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Summary

Card programmable calculators together with suitable programs can easily provide numerical answers which formerly took large computers and volumes of tabular or plotted outputs for everyday reference. Presented here are derivations and program listings for the HP-65 and HP-67 calculators to provide the user with commonly used radar detection performance data.

Both fixed threshold and adaptive threshold CFAR detection with noncoherent integration are covered. Recursive programs for the general chi-squared target fluctuation distribution are treated as well as faster running recursive programs for the Swerling target Cases I - IV. Also included are fast programs for the required detection signal-to-noise ratio based on the simplified algorithms of Barton.

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SECTION I

INTRODUCTION

The problem treated here is the classical detection of RF target signals in a Gaussian noise background. This was initially analyzed and presented by Marcum and Swerling[1,2] and extended, since that time by a number of writers. Since tabulated lists and curves are often somewhat awkward to use, the writers were interested in hand-calculator programs which could give numerical results over a wide variety of detection parameters. In particular, programs for HP-65 and HP-67 calculators are presented, but the algorithmic approaches could easily be programmed on other calculators.

The writer sta[illegible] this work by following Barton's[3] approach, an empirical approximation which directly provides required signal-to-noise ratio (SNR) for given probability of detection (PD) for various target models. The accuracy of the approximation is within a dB for normal parameter values, but the approach does not lend itself to finding probability of detection, given the SNR, without an iterative approach which is beyond the capability of the HP-65.

Later, the writer found an excellent report by Shnidman[4] who had found finite recursive series solutions for probability of detection for the basic four Swerling target models for fixed-threshold detection, and who also presented infinite series algorithms for nonfluctuating and the generalized chi-squared target models. These algorithms were found to be directly programmable for the HP-65. S. P. Applebaum has translated these programs for the HP-67 and these are included here. For the special case of a Swerling Case II target, these programs can solve for either PD or for SNR.

In spite of the importance of both CFAR detection and noncoherent integration, there are few papers in the literature which combine the two. One excellent paper, however, is that of Mitchell and Walker[5]. This paper treats the background estimation type of CFAR, and it has provided algorithms used here to cover these cases. For the special case of Swerling Case II, a finite series solution and programmed iterative inverse can provide either PD or SNR as with the fixed-threshold case. For other target models, a truncated infinite series solution provides PD given the SNR.

The implementations which these analyses treat are illustrated in Figures 1-1 and 1-2. In Figure 1-1, the noise background is considered to be constant and known. N samples of signal-plus-noise are summed and compared to a fixed threshold. If the threshold is exceeded, a target detection is declared. The threshold is set according to a specified false-alarm probability in the absence of signal. In the recursive solutions programmed here, the first step is the calculation of the threshold value given the false-alarm probability.

Figure 1-2 considers the case where the background noise is unknown or slowly varying so that a noise estimate must be made in order to establish a detection threshold value. The noise estimate here considered is the sum of R independent detected noise samples, and corresponds to the most common type of constant false-alarm rate (CFAR) detector.

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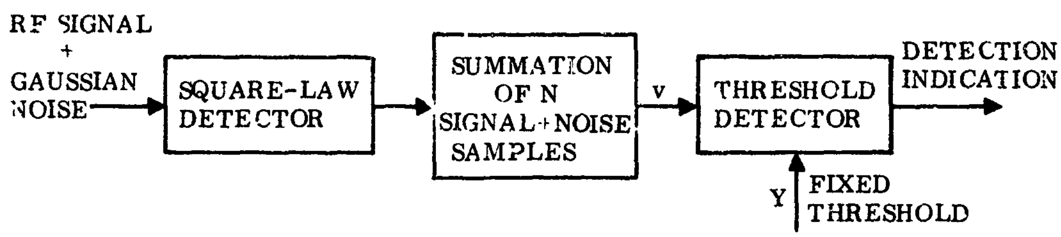

Figure 1-1. Fixed-Threshold Detection

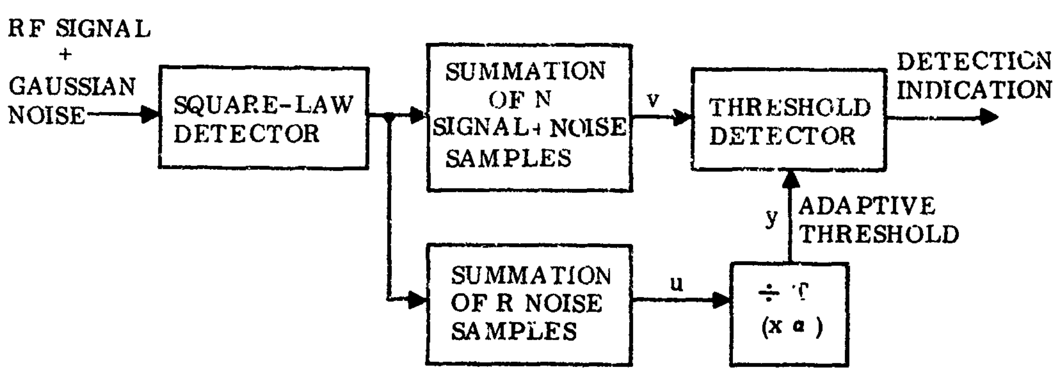

Figure 1-2. CFAR Detection (Background Normalized)

The target models used here follow that of Swerling and the integrated target SNR distributions all may be considered special cases of the chi-squared (or gamma) distribution given by

$$w(z, Z) = \frac{1}{(K-1)!} \left(\frac{K}{Z}\right)^K z^{K-1} e^{-\frac{Kz}{Z}} \tag{1-1}$$

where z is the integrated target power SNR for any trial and $Z = \bar{z}$. K is a distribution parameter which can have any value greater than zero and certain values of K correspond to standard target fluctuation cases* as follows:

$0 < K < 1$: Weinstock case

$K = 1$: Swerling case I

$$w(z, Z) = \frac{1}{Z} e^{-\frac{z}{Z}} \tag{1-2}$$

Exponential power or Rayleigh voltage distribution. Target constant over N integrated samples. Results from radar target composed of many separate scatterers. Often satisfied by aircraft.

$K = 2$: Swerling case III

$$w(z, z) = \frac{4}{Z^2} e^{-\frac{2z}{Z}}$$

Approximate distribution of target with Rayleigh and fixed components of equal average power. Target constant over N integrated samples.

$K = N$: Swerling case II

Same basic target distribution as for K = 1 but with target amplitude independent over N integrated samples. Often satisfied by aircraft targets with pulse-to-pulse radar frequency agility.

$K = 2N$: Swerling case IV

Same basic target distribution as for K = 2 but with target amplitude independent over N integrated samples.

---

* Nathanson[6] provides a good discussion of these target models.

$K = \infty$: Nonfluctuating target or case 0

$z = Z$

Note that other values of K may be useful for cases with different target fluctuation rates or with block correlation within the N samples integrated.[7] If the diversity order within the N samples is Ne then with a Rayleigh target K should be taken equal to Ne. For a Case III target distribution K should be taken as 2Ne.

SECTION II

FIXED THRESHOLD DETECTION - RECURSIVE SOLUTION

1. SWERLING CASE II

Although, as shown later, generalized programs can be written to cover chi-squared distributions of any K, it is worthwhile to consider some special cases since they can provide both simpler and faster running calculator programs. We shall start with the simplest of these cases - fixed threshold detection with a Swerling Case II target model.

The probability density function of v, the integrated signal-plus-noise variate, is given by, *

$$f(v) = \frac{v^{N-1}}{(1+X)^N (N-1)!} e^{-\frac{v}{1+X}} \tag{2-1}$$

where X is the average SNR of each sample. The probability of detection is then given by

$$P2 = \int_Y^\infty \frac{v^{N-1}}{(1+X)^N (N-1)!} e^{-\frac{v}{1+X}} dv \tag{2-2}$$

This may be integrated by parts to give

$$P2 = \sum_{m=0}^{N-1} \frac{Y^m}{m!(1+X)^m} e^{-\frac{Y}{1+X}} \tag{2-3}$$

Notice that for X = 0 this reduces to the false alarm probability (PF)

$$PF = \sum_{m=0}^{N-1} \frac{Y^m}{m!} e^{-Y} \tag{2-4}$$

* Eqn. III.10 of Swerling or Eqn. (39) for $f_N(V|X)$ of Mitchell and Walker on page 675 noting that our $X = Z/N$ corresponds to their X/N.

which provides an implicit solution for Y given PF. Notice also that by substituting $Y = Y/1+X$ in Equation (2-4), we obtain Equation (2-3) so that one program routine can be used for both. This common equation shall be written as

$$P = \sum_{m=0}^{N-1} \frac{Y^m}{m!} e^{-Y} \tag{2-5}$$

with each definition of Y giving the appropriate corresponding definition of P.

Given a desired PF, the first step in finding either P2 given X or X given P2 must be to find Y using Equation (2-5) with P = PF. Perhaps one's first thought might be to use Newton's method to find the root of P - PIN where PIN is the specified value and P is obtained from the equation for a given Y. However, since P versus Y has an inflection point, convergence is not assured and it is better to use ln (P/PIN). This leads to incrementing Y for successive trials by

$$\Delta Y = -\frac{\ln P - \ln PIN}{\frac{d}{dY}(\ln P)} = -\frac{P}{P'} \ln (P/PIN)$$

We find by differentiating Equation (2-5) that

$$P' = -\frac{Y^{N-1}}{(N-1)!} e^{-Y}$$

$$= - \text{last term of P series.}$$

In the algorithmic expressions, the m^{th} term of this series is used and shall be designated YM. Each term is determined recursively from the previous term. After completing the series, we will have in storage the last term, YM, so we can use this for - P'. Therefore, in applying Newton's method, the Y-increment for successive trials is given by

$$\Delta Y = \frac{P}{YM} \ln \frac{P}{PIN} \tag{2-6}$$

To begin the iteration of Newton's method a starting value for Y is also needed which shall be designated Y0. The writer found empirically that the following expression approximated Y quite closely for small values of PIN as may customarily be desired for PF, and was programmable with very few program steps on the HP-65.

$$Y0 = N - \sqrt{N} + 2.3\sqrt{L}\,(\sqrt{L} + \sqrt{N} - 1)$$

$$L = -\log \text{PIN} \qquad (2\text{-}7)$$

Using this start only three or four iterations are needed to calculate Y to 10 significant figures for any value of PIN of interest.

This solution of Equation (2-5) for Y can more completely be specified by using the algorithmic notation of Iverson (following Shnidman's practice) as given in Figure 2-1.

A brief explanation of this notation is first in order. The arrow notation implies a specification, that is, the statement, $L \leftarrow -\log \text{PIN}$, is translated to mean that the quantity L is specified by - log PIN. The normal execution of the statements is line by line starting at the top, but a branch may be designated by an arrow between two statement lines. A conditional branch is denoted by a colon statement, and the branch is executed if the comparison condition specified on the arrow is satisfied. Otherwise the next statement in the sequence is executed.

The brackets labeled D and E on Figure 2-1 correspond to subroutines in the HP-65 program which follows and are shown here for convenience. Notice that the iteration is terminated when $|\Delta Y/Y|$ is less than 10^{-6}. Since the stored value of Y has already been corrected by the indicated ΔY and the convergence is quite rapid, Y is usually accurate to 9 or 10 significant figures.

Having obtained Y for a given PF using this algorithm, we can calculate P2 directly from Equation (2-5) [i.e., subroutine E] by the substitution Y = Y/(1+X). Alternatively, if P2 is given and it is desired to find X, we can substitute PIN = P2, find a corresponding Y2 using the program of Figure 2-1 an then find X = Y/Y2 - 1.

These features are all contained in the Program HP-65 Y-P2 given here. Most of the program comes directly from the algorithmic program of Figure 2-1, but a few comments may help in its understanding. First of all, the writer has often recorded in the comment space on each line of the HP-65 programs the stack contents in the order, x, y, z, t. This may be useful to understanding since the stack is often used in these programs for temporary storage. This practice saves on use of the storage registers which is sometimes necessary and also often leads to shorter, faster running programs.

```
L ← - log PIN
Y0 ← N - √N + 2.3 √L (√L + √N - 1)

Y ← Y0

YM ← e^-Y
YMS ← YM
M ← 0
M ← M+1

=
N : M

YM ← YM x Y/M
YMS ← YMS + YM

P ← YMS

ΔY ← (P/YM) ln (P/PIN)
Y ← Y + ΔY

≤
10^-6 : |ΔY/Y|

EXIT
```

D

E

Figure 2-1. Algorithmic Program for Y Given P

Next, the LBLE subroutine incorporates LBLD as a loop to save a program step. When the D statement is reached on the last step, it jumps to LBLD. When the test $N \leq M$ is satisfied, the RTN jumps back to the E-subroutine call, since the HP-65 has only one program step register, and that holds only the initial subroutine call step number no matter how many successive subroutines are called before the RTN statement is executed.

Since for finding X given PF and P2, the Y-algorithm programmed here is used twice, it is preceded by storing the previously calculated Y from R2 in R6. The first time through nothing exists in R2 anyway, so these steps can be ignored in trying to understand the program. After running the second time with PIN ← P2 the program stops with Y2 displayed. Depressing the R/S key then restores Y in R2 and calculates X from the Y2 and Y previously found.

Finding P2 given X uses the LBLA function and is straightforward, requiring only that Y has been previously calculated or otherwise stored in R2.

It is interesting to note that Shnidman was concerned about accuracy of the calculation and underflow for certain cases such as e^{-Y} for large values of Y. His computer was equivalent to about 7 digit words and he went to double-precision arithmetic and logarithmic calculation in underflow cases. With the 10 digit words and $10^{\pm 99}$ range of the HP calculators, together with direct monitoring by the operator, such measures are really unnecessary. The programs have been written such that if input parameters which would lead to underflow are entered, the underflow condition results almost immediately. This is indicated on the HP-65 by interruption of the program sequence with the display reading zero.

2. SWERLING CASES I, III, IV

In this section, the basic expressions to be programmed will not be derived but will be taken directly from Shnidman[2] to which the reader is referred for more detail.

We shall refer to the function represented by Equation (2-5) as P (N, Y). The probability of detection of a Swerling Case I target can then be found as

$$P1 = e^{-Y/(Z+1)} \qquad \text{for } N = 1,$$

or

$$P1 = P(N-1,\ Y) + \left(\frac{Z+1}{Z}\right)^{N-1} e^{-Y(Z+1)} \left[1 - P\left(N-1,\ \frac{YZ}{Z+1}\right)\right] \quad \text{for } N \geq 2 \qquad (2\text{-}8)$$

Here the integrated power signal-to-noise ratio, Z = NX was used.

The HP-65 P1 program directly implements this expression. Prior to running this, the Y-P2 program must be run to store Y in the R2 register. The same basic LBL E-subroutine is used here as in the prior program, modified slightly to give P(N-1, Y1) where Y1 is in the X-register prior to calling the subroutine. The coefficient of [1 - P] is also tested so that if too small, the LBLE subroutine is not run a second time.

In a similar way, the PD for a Swerling Case III target can be found as

$$P3 = e^{-\frac{2Y}{Z+2}} \left[1 + \frac{2YZ}{(Z+2)^2}\right] \qquad \text{for } N = 1$$

or

$$P3 = \frac{Y^{N-2} e^{-Y}}{(N-2)!} \cdot \frac{2Y}{Z+2} + P(N-1, Y)$$

$$+ \left(\frac{Z+2}{Z}\right)^{N-2} e^{-\frac{2Y}{Z+2}} \left[1 - \frac{2(N-2)}{Z} + \frac{2Y}{Z+2}\right] \cdot \left[1 - P(N-1, \frac{YZ}{Z+2}\right] \quad \text{for } N \geq 2$$

(2-9)

The HP-65 P3 program directly implements this expression. It is run following the Y-P2 program to find Y as was the P1 program.

Shnidman shows that the PD for a Swerling Case IV target can be written as

$$P4 = \sum_{M=0}^{N-1} \frac{V^M}{M!} e^{-V} + \sum_{M=N}^{2N-1} \frac{V^M}{M!} e^{-V} \left[1 - \sum_{K=0}^{M-N} \frac{N!}{K!\,(N-K)!} \left(\frac{X}{X+2}\right)^K \left(\frac{2}{X+2}\right)^{N-K}\right]$$

(2-10)

where

$$V = 2Y/(X+2).$$

Although the first term is clearly equal to P(N, V), the second term is an extended summation of the same form as the first with a more complex term-by-term multiplier. The programming involves a doubly recursive approach which is best illustrated in algorithmic form on Figure 2-2.

```
M ← 0
ZK ← [2/(X+2)]^N
V ← 2Y/(X+2)

YM ← e^-V
YMS ← YM
M ← M+1

≤
N : M

YM ← YM  V/M
YMS ← YMS + YM

SUM ← YMS
ZKS ← ZK

YM ← YM · V/M
SUM ← SUM + YM (1 - ZKS)
ZK ← ZK · X/2 · (2N - M)/(M - N+1)
ZKS ← ZKS + ZK
M ← M+1

≤
2N : M

P4 ← SUM  ──→ EXIT
```

Figure 2-2. Algorithmic Program for P4

For relating Figure 2-2 to Equation (2-10), note that K = M-N so that no separate index is needed. The nomenclature YM corresponds to $(V^M/M!)\, e^{-V}$ while ZK corresponds to each term of the K summation and YMS and ZKS have corresponding relationships to YM and ZK, respectively.

The order of some of the steps listed here is arbitrary and are written to correspond to the program HP-65 P4 for consistency. This program is used, as for P1 and P3 after running Y-P2 to find Y.

3. GENERALIZED AND NONFLUCTUATING TARGET MODEL

For the nonfluctuating target, as well as the general case, the summation of an infinite series is required, and the nonfluctuating target can be considered a special case of the general formulation. Mitchell and Walker give a straightforward derivation which in our nomenclature can be written as follows.

The distribution of v for a given integrated signal-to-noise ratio, z, is given by

$$f_N(v \mid z) = \left(\frac{v}{z}\right)^{\frac{N-1}{2}} e^{-(v+z)}\, I_{N-1}(2\sqrt{vz})$$

$$= \sum_{b=0}^{\infty} \frac{z^b\, v^{N+b-1}\, e^{-(v+z)}}{b!\,(N+b-1)!} \qquad (2\text{-}11)$$

as given by Marcum and Swerling.

The probability that v will exceed a fixed threshold, Y, is then

$$P(v > Y \mid z) = \int_Y^{\infty} f_N(v \mid z)\, dv$$

$$= \sum_{b=0}^{\infty} \frac{z^b}{b!}\, e^{-z} \sum_{m=0}^{N+b-1} \frac{Y^m}{m!}\, e^{-Y} \qquad (2\text{-}12)$$

For a nonfluctuating target, this gives the desired PD by letting z = Z. For a fluctuating target, we must integrate over the distribution of z as follows

$$P = \int_0^\infty w(z, Z)\, P(v > Y \mid z)\, dz \qquad (2\text{-}13)$$

Using w(z, Z) for the generalized chi-squared distribution of Equation (1-1), this yields the generalized PD,

$$PG = \sum_{b=0}^{\infty} \frac{(K+b-1)!}{b!\,(K-1)!} \left(\frac{K}{K+Z}\right)^K \left(\frac{Z}{K+Z}\right)^b \sum_{m=0}^{N+b-1} \frac{Y^m}{m!}\, e^{-Y} \qquad (2\text{-}14)$$

Shnidman changes the order of summation of these expressions so as to get a more direct measure of error which can be used to truncate the summation to a finite number of terms. This expression then becomes

$$PG = \sum_{m=0}^{N-1} YM + \sum_{M=N}^{\infty} YM \left(1 - \sum_{b=0}^{M-N} XB\right)$$

where

$$YM = \frac{Y^M}{M!}\, e^{-Y}$$

and (2-15)

$$XB = \frac{(K+b-1)!}{b!\,(K-1)!}\,(1-V)^K\, V^b, \quad V = \frac{Z}{K+Z}$$

Note that a nonfluctuating target corresponds to the limit as $K \rightarrow \infty$ for which

$$XB \rightarrow \frac{Z^b}{b!}\, e^{-Z} \qquad (2\text{-}16)$$

The error in Equation (2-15) for a truncated summation is shown by Shnidman to be given by the product,

$$\epsilon_M = \left(1 - \sum_{m=0}^{M} YM\right)\left(1 - \sum_{b=0}^{M-N} XB\right) \qquad (2\text{-}17)$$

The programs for PG given here test this product after each term of the PG summation and when it becomes less than 10^{-8}, which seems a suitably small number, the summation is stopped.

An algorithmic program for PG is given in Figure 2-3, the program corresponding directly to the HP-65 PG program. As with the other programs, it is necessary to run Y-P2 first to find Y. The initiation of the PG program requires entry of both X, and the target distribution parameter, K.

Suitable values of K were discussed in the introduction, but the special case of a nonfluctuating target provides some difficulty since infinity is not an allowable entry value. The best large number to substitute for infinity in this case was found to be about 10^5 (entered with only two keystrobes as EEX 5). Larger values for K give difficulties for some values of Z = NX in calculating the initial value of XB, while smaller values are less accurate approximations of infinity. This compromise, however, apparently gives an accuracy for the calculated P0 of at least three places for any value of Z. To avoid the required entry of K in this case, as well as to provide greater accuracy if wanted, a modified form of the PG program is given here as the P0 program which calculates the detection probability for a nonfluctuating target. This is based on using Equation (2-16) for XB in place of the more general Equation (2-15). This eliminates the computation difficulty for a large value of K, since it is not used.

```
YM ← e^-Y
YMS ← YM
M ← 1

            ≤
        N : M

YM ← YM·Y/M          }
YMS ← YMS + YM       } A
M ← M+1              }

SUM ← YMS
V ← Z/(K+Z)
XB ← (1-V)^K
ZK1 ← K
XBS ← XB

YM ← YM Y/M                }
YMS ← YMS + YM             } A
SUM ← SUM + YM (1-XBS)     }
M ← M+1
XB ← XB·V·ZK1/(M-N)
ZK1 ← ZK1+1

            <
        ε : (1-YMS) (1-XBS)

P ← SUM ────────► EXIT
```

Figure 2-3. Algorithmic Program for PG

SECTION III

FIXED THRESHOLD DETECTION - BARTON ALGORITHM

Barton[3] and Cann[8] were interested in a somewhat universal set of curves which could be used simply to find radar detection performance over various target and radar parameters. They found that an ideal detector curve plus a set of relatively simple loss factors, i.e., detector loss, integration loss, collapsing loss, and fluctuation loss, gave very reasonable accuracy for normal values of PD, PF, and N, and for the target distributions we have been considering. Barton's algorithms have also been programmed for the HP-65 and HP-67 and are included here. These programs are complementary to the PD programs previously given in that they calculate a required signal-to-noise ratio for a given PD rather than the other way around.

For the nonfluctuating target, and no collapsing ratio, the Barton/Cann algorithm can be written

$$\mathrm{SNR\ (dB)} = 10 \log \left\{ \frac{1}{2} \left[X + \sqrt{X(X+9.2)} \right] \right\}$$

$$X = \frac{1}{2N} \left[Q^{-1}(PF) + Q^{-1}(PD) \right]^2$$

$$Q(y) = \frac{1}{2\sqrt{\pi}} \int_Y^{\infty} e^{-\frac{t^2}{2}} dt \qquad (3\text{-}1)$$

(To correlate this with Barton's nomenclature $X_o = 2X$)

The inverse Q function is calculated by the approximation.[9] For

$$P \leq \frac{1}{2},\ Q^{-1}(P) = t - \frac{a_o + a_1 t}{1 + b_1 t + b_2 t^2} \qquad (3\text{-}2)$$

where

$$a_0 = 2.31,$$

$$a_1 = 0.271,$$

$$b_1 = 0.992,$$

$$b_2 = 0.0448,$$

$$t = \sqrt{\ln (1/P^2)} ,$$

and for $P > 1/2$, $Q^{-1}(P) = -Q^{-1}(1 - P)$.

Also, integration gain is given by:

$$Gi = SNRN - SNR1$$

These are directly programmed in the HP-65 SNRN and HP-67 SNR programs.

For a nonfluctuating target with collapsing loss, or for a fluctuating target, the HP-65 SNRF and HP-67 SNR programs make the following calculation:

$$SNRF\ (dB) = SNRN + \log \frac{N}{N_s} + Lf , \tag{3-3}$$

where Lf, the fluctuation loss, is given by

$$Lf(dB) = \frac{10 \log D - SNR1}{N_s} \tag{3-4}$$

SNRN is the value calculated by program SNRN for N and SNR1 is the value calculated by program SNRN for N = 1. D is the single pulse average SNR for fluctuating target detection and depends on the target model used. For the Rayleigh fluctuation model of Swerling's Cases 1 and 2

$$D_{12} = \frac{\ln PF}{\ln PD} - 1 \tag{3-5}$$

For the one dominant plus Rayleigh fluctuation model of Swerling's Cases III and IV, D_{34} is given implicitly by[10]

$$PD = \left(1 - \frac{2\,D_{34}\ln PF}{(2+D_{34})^2}\right) PF^{\frac{2}{2+D_{34}}} \tag{3-6}$$

The writer found that the solution to this equation is well approximated by

$$D_{34} = (0.361 - \log PD)\left(\frac{3.27}{\sqrt{1-PD}} - 1.29 - 0.96\sqrt{1-PD}\right) - 2 \tag{3-7}$$

and this expression is used in HP-65 SNRF and HP-67 SNR to avoid the need for reiteration. The greatest error in this approximation occurs for low values of PD but it is accurate to better than 0.5 dB for PD equal 50% and within 1 dB for PD equal 30%. For PD greater than 90%, it is accurate to within 0.2 dB. An extremely bad choice of PD too low, or PF too large may cause D_{34} to be negative and flashing zeros will indicate this error when running the program.

Finally, the programs calculate the diversity gain as

$$Gd\,(dB) = (N_e - 1)\,Lf$$

and the range ratio, re Swerling

$$R/R_o = 10^{SNRF/40}$$

SECTION IV

CFAR DETECTION - RECURSIVE SOLUTION

1. SWERLING CASE II

As with the fixed threshold, the Case II target model leads to a simple analysis and finite summation for finding the probability-of-detection. Starting with Equation (2-3), the constant Y can be replaced by the variable y to have

$$P(v > y) = \sum_{m=0}^{N-1} \frac{\left(\frac{y}{X+1}\right)^m}{m!} e^{-\frac{y}{X+1}} \tag{4-1}$$

for each specific value of y.

Note on Figure 1-2 that y is derived from u and that u is the sum of R independent Rayleigh noise samples of unit average power - unity since we also normalized the magnitude of v to the average noise power. Therefore, u has the distribution

$$p(u) = \frac{u^{R-1}}{(R-1)!} e^{-u} \tag{4-2}$$

Then the overall probability of v exceeding y is given by

$$P = \int_0^\infty p(u)\; P\left(v > \frac{u}{T}\right) du \tag{4-3}$$

where T is a calibrating factor which must be set to achieve the desired false-alarm probability and is analogous in our further derivation here to Y which determined the false-alarm probability in the fixed threshold case. Substituting Equations (4-1) and (4-2) into Equation (4-3), interchanging the order of summation and integration, and integrating, one gets

$$P = \sum_{m=0}^{N-1} P_m = \sum_{m=0}^{N-1} \frac{(R+m-1)!}{m!\,(R-1)!} \frac{(T2)^R}{(T2+1)^{R+m}} \tag{4-4}$$

where $T2 = T(X+1)$.

In a similar manner to that for finding Y previously, let X = 0 so that T2 = T and find the value of T for which P equals the false-alarm probability.

This process is best done by using Newton's method on ln (P/PIN) as before so that

$$\Delta T = \frac{P \ln(P/PIN)}{\frac{dP}{dT}} \tag{4-5}$$

and we find from Equation (4-4) that

$$\frac{dP}{dT} = \frac{R}{T} \sum_{m=0}^{N-1} P_m - \sum_{m=1}^{N} m\, P_m \tag{4-6}$$

Denoting the last summation as Q, Equations (4-4), (4-5) and (4-6) yield

$$\Delta T = \frac{\ln(P/PIN)}{Q/P - R/T} \tag{4-7}$$

Since the terms of Q are closely related to those of P, both sums can be formed at the same time.

We are left with the problem of the initial value to use for T. Extending the curve fitting approach of before the writer found a reasonable initial value to be given by

$$\frac{1}{T0} = \frac{1 - (PIN)^{1/R}}{(PIN)^{1/R}} \left[(1-B)\,N + B \left(\frac{N - \sqrt{N}}{2.3L} + \frac{\sqrt{L} + \sqrt{N} - 1}{\sqrt{L}} \right) \right]$$

$$B = \frac{R-1}{R+0.922}, \qquad L = -\log PIN \tag{4-8}$$

The value of T0 from Equation (4-8) was found to provide a sufficiently good start for iterative convergence over the range of $10^{-10} < P < 1$ and $1 < R < 1000$.

Unfortunately, this takes more than 50 program steps so that a separate program card is necessary for data entry and calculation of T0 with the HP-65. After running this program, HP-65 P2C can be used to perform the iterative calculation of T for a given false-alarm probability and P2 for various input values of SNR = 10 log X. If SNR is to be found for a given P2 program, HP-65 T0 must be rerun with P2 input, followed by HP-65 P2C again, the process being directly analogous to that of the HP-65 Y-P2 program for a fixed threshold. Since the HP-67 has more program storage, the T0 calculation is included in the HP-67 P2C program.

2. GENERALIZED TARGET MODEL

The relationship of the CFAR process to be fixed threshold process in general is the same as it was for Case II. Starting with Equation (2-14), let $Y = u/T$ and integrate over the distribution of u from Equation (4-2) to find the overall PD. By this process, one obtains

$$\mathrm{PGC} = \sum_{b=0}^{\infty} \mathrm{XB} \sum_{m=0}^{N+b-1} \mathrm{PM}$$

or

$$\mathrm{PGC} = \sum_{m=0}^{N-1} \mathrm{PM} + \sum_{m=N}^{\infty} \mathrm{PM} \left[1 - \sum_{b=0}^{m-N} \mathrm{XB} \right]$$

where (4-9)

$$\mathrm{XB} = \frac{(k+b-1)!}{b!\,(K-1)!} (1-V)^K V^b, \qquad V = \frac{Z}{K+Z}$$

and

$$\mathrm{PM} = \frac{(R+M-1)!}{m!\,(R-1)!} (1-A)^R A^m, \quad A = \frac{1}{T+1} .$$

Note that XB is the same as used for the fixed threshold case and for the nonfluctuating case

$$\mathrm{XB} \rightarrow \frac{Z^b}{b!} e^{-Z}$$

Similarly, the fixed threshold case it approached by letting $R \rightarrow \infty$ so that $y = \frac{u}{T} \rightarrow \frac{R}{T} = Y$ so that

$$A \rightarrow \frac{Y}{Y+R}$$

and

$$\mathrm{PM} \rightarrow \frac{Y^m}{m!} e^{-Y}$$

An algorithmic program for PGC is given in Figure 4-1 which follows closely that for PG. Unfortunately, this could not be fitted into 100 HP-65 program steps so it had to be programmed on two cards, HP-65 PGC(1) and PGC(2). PGC(1) incorporates the iteration for T given T0 and finds A = 1/(T+1). Therefore, PGC is found by running in sequence HP-65 T0 to find T0, PGC(1) to find A and to enter SNR and K, and finally by running PGC(2) as noted on the CFAR Detection instruction sheet. For the HP-67, the entire calculation is included on the single program card, HP-67 PGC.

3. CFAR LOSS

The increase in SNR required with a CFAR detector, as compared to a fixed threshold detector, has been termed CFAR loss. This concept is a convenient one because the CFAR loss is essentially independent of the target fluctuation model, at least as far as the five Marcum and Swerling models are concerned. Although many papers in the literature deal with CFAR loss for various CFAR detector schemes, the paper by Mitchell and Walker[5] is the only one found by the writer to cover the combination of noncoherent signal integration with a background normalizer threshold. Using the HP-65 Programs of this paper for Case II targets, the data of Figures 4-2 through 4-5 were calculated and are presented here for convenience. The loss values from these curves may be used as a correction to fixed threshold SNRs for the fixed threshold programs. This may be handier than running the CFAR programs for many cases because of the long running time of the generalized CFAR program. Although the writer has verified in a few sample cases that other target models and other detection probabilities give essentially the same CFAR loss values, it will be left as an exercise for the reader to be convinced that this is true for the cases of concern.

$$PM \leftarrow (1-A)^R$$

$$PMS \leftarrow PM$$

$$M \leftarrow 1$$

$$\leqslant$$

$$N : 1$$

$$\left.\begin{aligned} PM &\leftarrow PM \cdot \frac{R+M-1}{M} \cdot A \\ PMS &\leftarrow PMS + PM \\ M &\leftarrow M+1 \end{aligned}\right\} A$$

$$V \leftarrow Z/(K+Z)$$

$$XB \leftarrow (1-V)^K$$

$$XBS \leftarrow XB$$

$$SUM \leftarrow PMS$$

$$\left.\begin{aligned} PM &\leftarrow PM \ \frac{R+M-1}{M} \cdot A \\ PMS &\leftarrow PMS + SUM \\ M &\leftarrow M+1 \end{aligned}\right\} A$$

$$SUM \leftarrow SUM + PM\,(1-XBS)$$

$$XB \leftarrow XB\ \frac{K+M-N-1}{M-N} \cdot V$$

$$XBS \leftarrow XBS + XB$$

$$\leqslant$$

$$\epsilon : (1-XBS)(1-PMS)$$

$$P \leftarrow SUM \longrightarrow EXIT$$

Figure 4-1. Algorithmic Program for PGC

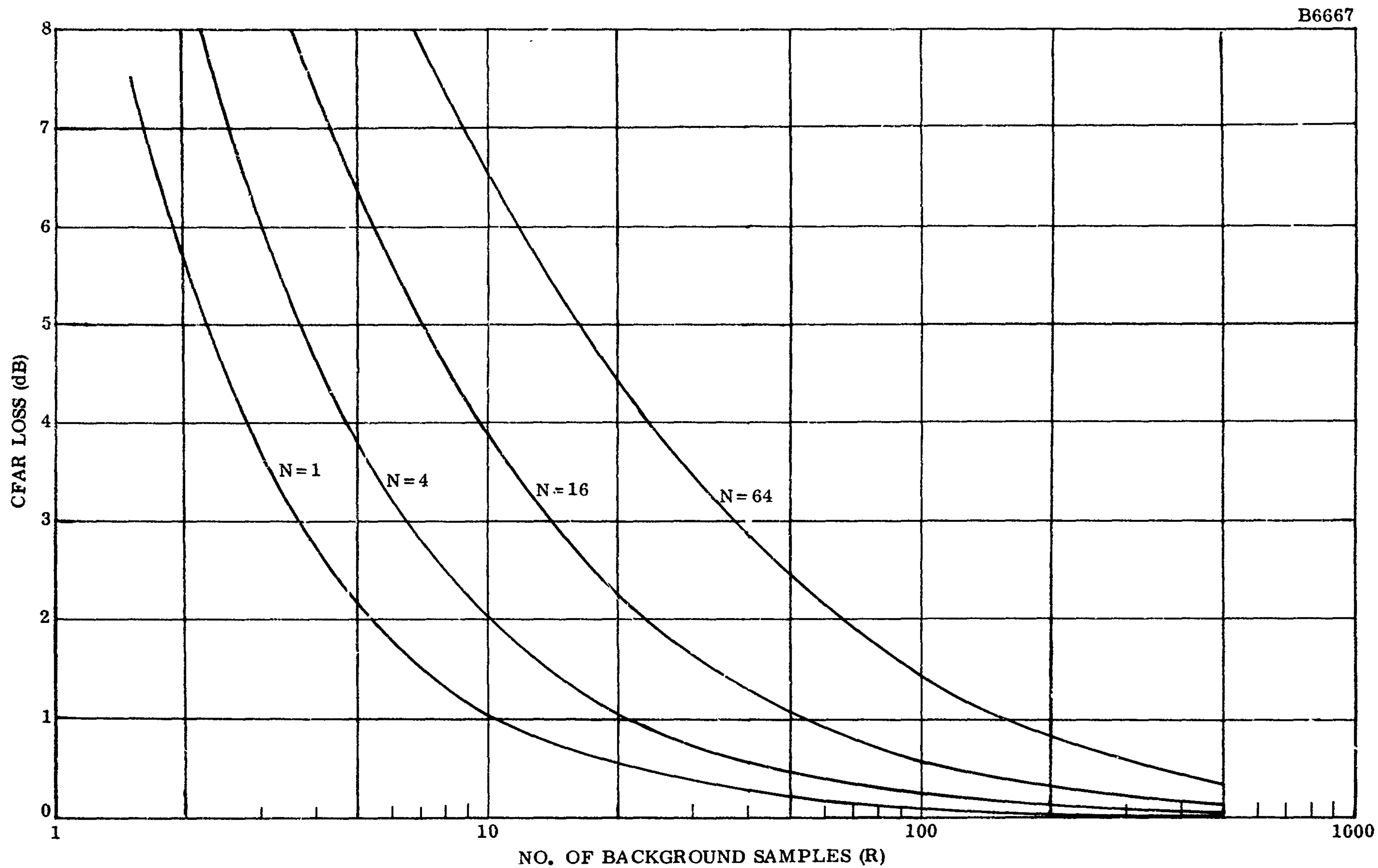

Figure 4-2. Square-Law CFAR Loss Swerling Case II Target, PF = 10^{-2}, PD = 0.5

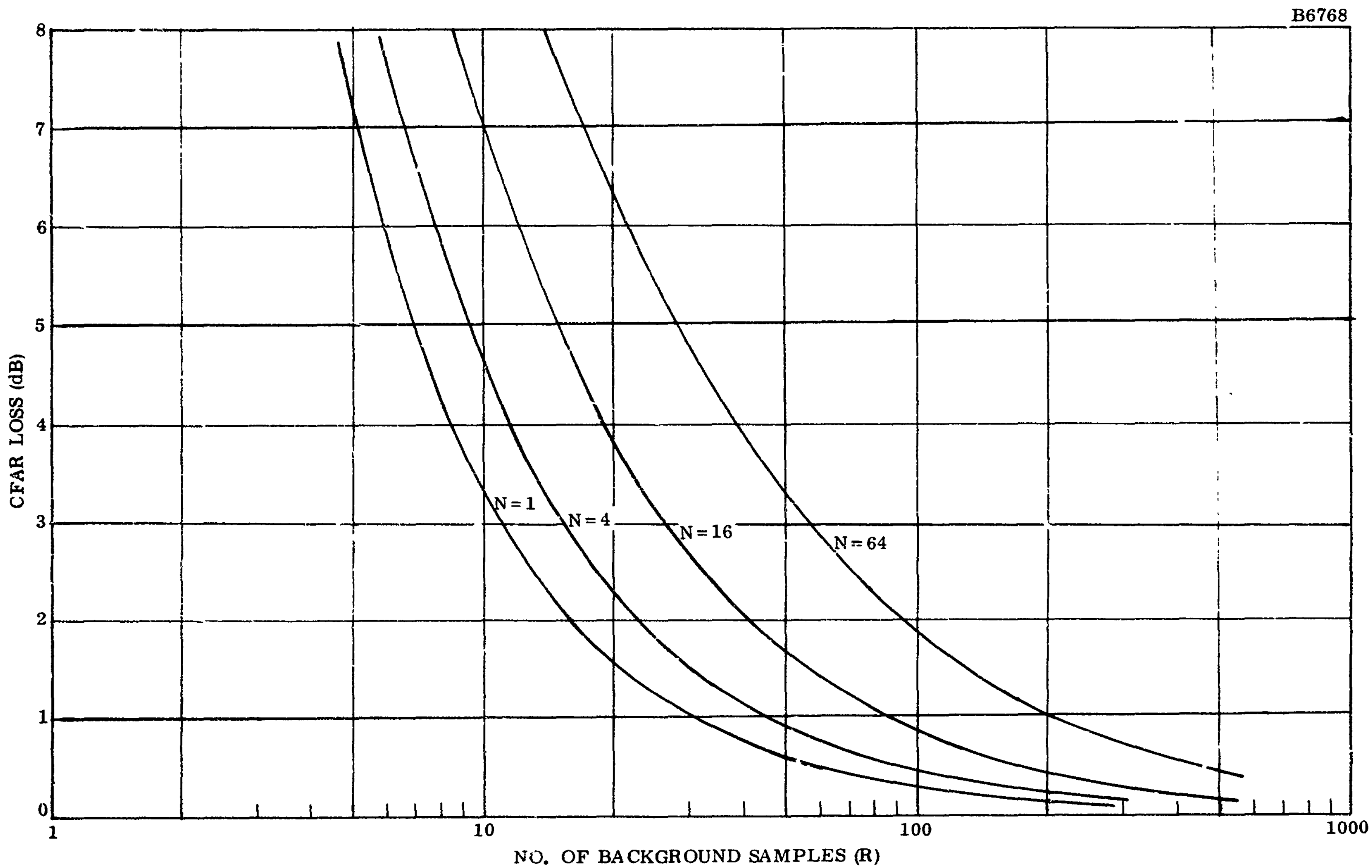

Figure 4-3. Square-Law CFAR Loss Swerling Case II Target, PF = 10^{-6}, PD = 0.5

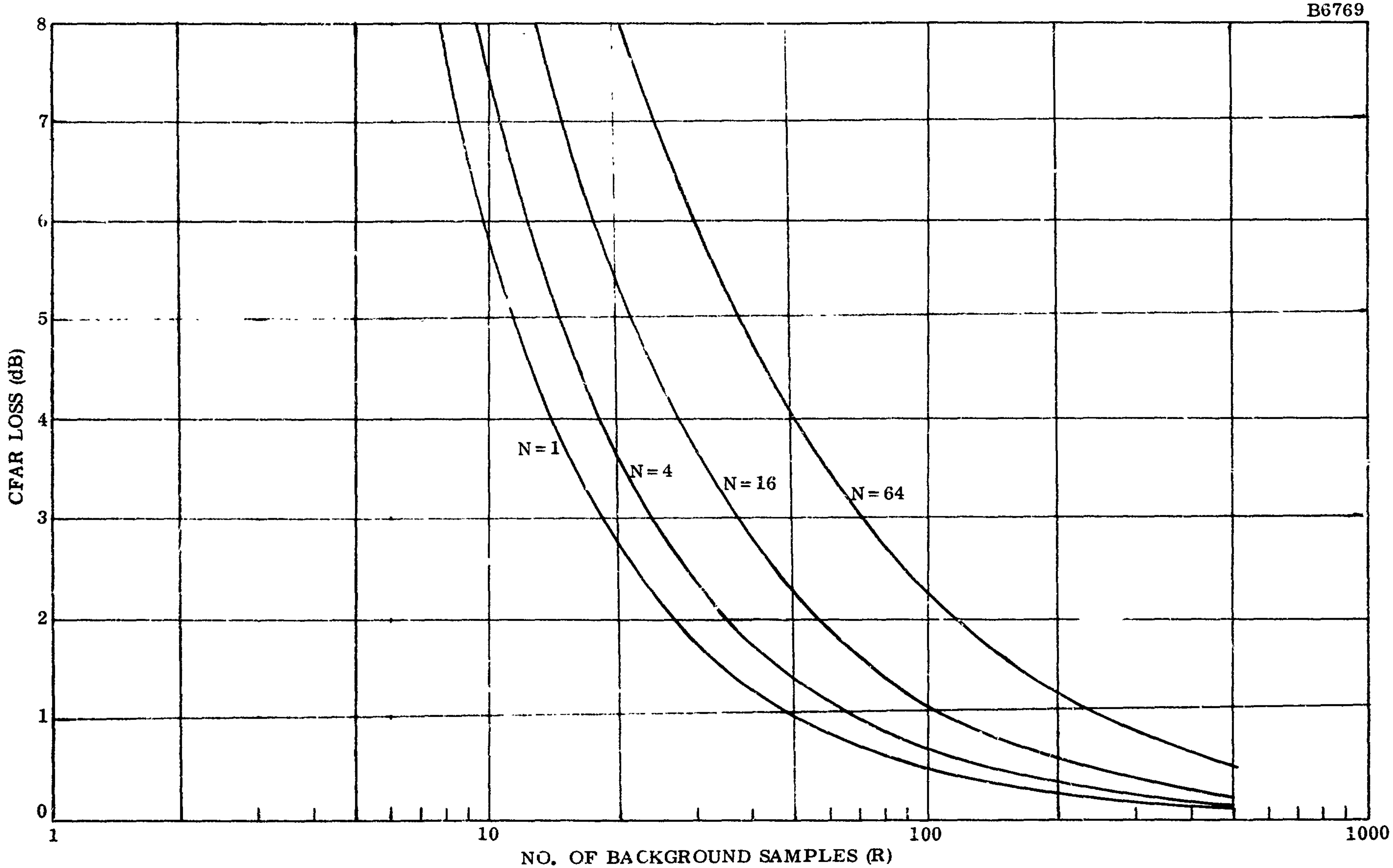

Figure 4-4. Square-Law CFAR Loss Swerling Case II Target, PF = 10^{-10}, PD = 0.5

SECTION V

HP-65 PROGRAMS

1. FIXED-THRESHOLD, RECURSIVE SOLUTIONS

These HP-65 programs calculate the probability of detection, given the number of samples noncoherently integrated, the false-alarm probability, and the average sample signal-to-noise ratio, for the various Swerling target models. The PG program does this for the generalized chi-squared target model. The Y-P2 program must be used to calculate the threshold value, Y, before using any of the other programs, and in addition it can calculate for a Case II target, either probability of detection given average signal-to-noise ratio or average signal-to-noise ratio given probability of detection.

Specific user instructions are as follows:

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS		OUTPUT DATA/UNITS
1	Enter program Y-P2	N	STO	1	
		PF	A		Y
	Go to step 2, 3, 4 or 6				
2	For P2 (Repeat or go to step 3 as desired)	X	B		P2
3	For X given P2	P2	A		Y2
			R/S		X
	Repeat or go to step 2, 4, or 6 as desired				
4	For P0, P1, P3, or P4 enter that program				
5		X	B		D
	Repeat or go to step 4 or enter Y-P2 and go to				
	step 2 as desired				
6	For general target model enter program PG				
7		K			
		X	B		P
	Repeat or go to step 4 or enter Y-P2 and go to				
	step 2 as desired				

K : Chi-squared distribution parameter
N : Signal and/or noise samples integrated
Ne : Target diversity within N samples
PF : Probability of false alarm
P : Probability of detection for chi-squared target
P0 : Probability of detection for nonfluctuating target
P1-P4 : Probability of detection for Cases I-IV
X : Avg sample power S/N within N samples
Y : Fixed detection threshold
Y2 : Y/(1+X)

Case 0 : $K=10^5$ (~3 place acc)
Case I : K=1
Case II : K=N
(Genl Rayleigh target : K=Ne)
Case III : K=2
Case IV : K=2N
(Genl Rayleigh + equal constant target : K=Ne)
Weinstock : 0 < K < 1

HP-65 Program Form

Title HP-65 Y-P2 Program Listing Page ___ of ___

SWITCH TO W PRGM PRESS f PRGM TO CLEAR MEMORY

KEY ENTRY	CODE SHOWN	COMMENTS
STO4	3304	PIN
RCL2	3402	SAVE Y IN R6
STO6	3306	
RCL4	3404	PIN
f	31	
LOG	08	
CHS	42	L
RCL1	3401	N
↑	41	
f	31	
√	09	√N, N, L
-	51	N - √N, L
gLSTX	3500	√N
1	01	
-	51	√N-1, N - √N, L→
gR↑	3509	L
f	31	
√	09	√L
+	61	√L + √N-1, N - √N
gLSTX	3500	√L
x	71	
2	02	
.	83	
3	03	
X	71	2,3 √L(√L+√N-1, N-√N
+	61	
STO2	3302	Y0
LBL	23	
1	01	
E	15	P
gR↑	3509	YM
÷	81	P/YM
RCL8	3408	P
RCL4	3404	PIN
÷	81	P/PIN, P/YM
f	31	
LN	07	
x	71	ΔY←(P/YM) LN(P/PIN)
STO	33	
+	61	
2	02	(Y ← Y+ΔY)
RCL2	3402	Y
÷	81	
g	35	
ABS	06	|ΔY/Y|
EEX	43	
CHS	42	
6	06	-6
gx≤y	3522	10 ≤ |ΔY/Y|
GTO	22	

KEY ENTRY	CODE SHOWN	COMMENTS
1	01	
RCL2	3402	Y
R/S	84	
RCL6	3406	RESTORE Y IN R2
STO2	3302	
gx↔y	3507	Y2, Y
÷	81	
1	01	
-	51	
R/S	84	X ← Y/Y2 - 1
LBL	23	
B	12	X
1	01	
+	61	
RCL2	3402	SAVE Y IN R6
STO6	3306	
gx↔y	3507	
÷	81	Y2 ← Y/(1+X)
STO2	3302	
E	15	P2
RCL6	3406	RESTORE Y IN R2
STO2	3302	
gR↓	3508	
R/S	84	P2
LBL	23	
E	15	
RCL2	3402	Y
CHS	42	
f^{-1}	32	
LN	07	YM ← e^{-Y}
STO8	3308	YMS ← YM
0	00	M ← 0
LBL	23	
D	14	M, YM
1	01	
+	61	M ← M+1
RCL1	3401	N
gx≤y	3522	N ≤ M
RCL8	3408	P ← YMS
RTN	24	
RCL2	3402	Y, N, M, YM
gR↑	3509	YM, Y, N, M
x	71	YM, Y, N, M→
gR↑	3509	M, YM·Y, N, M
÷	81	YM ← YM · Y/M
STO	33	
+	61	
8	08	(YMS ← YMS + YM)
gR↑	3509	
D	14	M, YM

REGISTERS	
R_1	N
R_2	Y
R_3	
R_4	PIN
R_5	
R_6	Y
R_7	
R_8	YMS
R_9	

LABELS	
A	
B	X→ P2
C	
D	YM Loop
E	Y→P
0	
1	
2	
3	
4	
5	
6	
7	
8	
9	

FLAGS	
1	
2	

TO RECORD PROGRAM INSERT MAGNETIC CARD WITH SWITCH SET AT W/PRGM

HP-65 Program Form

Title: HP-65 P0 Program Listing Page ____ of ____

SWITCH TO W/PRGM PRESS [f] [PRGM] TO CLEAR MEMORY

KEY ENTRY	CODE SHOWN	COMMENTS
RCL1	3401	N { X
x	71	B
STO3	3302	Z
RCL2	3402	
CHS	42	
f^{-1}	32	
LN	07	
STO7	3307	YM ← e^{-Y}
STO8	3308	YMS ← YM
1	01	
STO6	3306	M ← 1
LBL	23	
1	01	
RCL1	3401	N, M, YM
gx<y	3522	
GTO	22	
2	02	{ YM ← YM · Y/M
A	11	YMS ← YMS + YM
1	01	
RCL6	3406	
+	61	
STO6	3306	M ← M+1
GTO	22	
1	01	
LBL	23	
2	02	
RCL8	3408	
STO5	3305	SUM ← YMS
RCL3	3403	
CHS	42	
f^{-1}	32	
LN	07	
STO4	3304	XB ← e^{-Z}
gR↓	3508	XBS ← XB
LBL	23	
3	03	
gR↑	3509	XBS { YM ← YM · Y/M
A	11	YMS ← YMS + YM
gR↓	3508	XBS
CHS	42	
1	01	
+	61	
RCL7	3407	
X	71	YM(1-XBS), XBS→
STO	33	
+	61	
5	05	(SUM←SUM+YM(1-XBS))
gR↑	3509	XBS
RCL4	3404	XB
RCL3	3403	Z

KEY ENTRY	CODE SHOWN	COMMENTS
X	71	
RCL6	3406	
1	01	
+	61	
STO6	3306	M ← M+1
RCL1	3401	
-	51	K ← M-N
÷	81	
STO4	3304	XB ← XB · Z/K
+	61	XBS ← XBS + XB
↑	41	
CHS	42	
↑	41	
1	01	
+	61	1 - XBS
RCL8	3408	YMS
CHS	42	
1	01	
+	61	
x	71	(1 - YMS)(1 - XBS), XBS→
EEX	43	
CHS	42	
9	09	
gx≤y	3522	$10^{-9} \leq$ (1-YMS)(1-XBS)
GTO	22	
3	03	
RCL5	3405	
R/S	84	P0 ← SUM
LBL	23	
A	11	R
RCL7	3407	YM
RCL2	3402	Y
RCL2	3406	M, Y, YM, R
÷	81	
x	71	
STO7	3307	YM ← YM · Y/M, R→
STO	33	
+	61	
8	08	(YMS ← YMS + YM)
RTN	24	

REGISTERS	
R_1	N
R_2	Y
R_3	Z
R_4	XB
R_5	SUM
R_6	M
R_7	YM
R_8	YMS
R_9	Used

LABELS	
A	YM, YMS
B	
C	
D	
E	
0	
1	
2	
3	
4	
5	
6	
7	
8	
9	

FLAGS	
1	
2	

HP-65 Program Form

Title HP-65 P1 Program Listing Page ____ of ____

SWITCH TO W/PRGM PRESS f [PRGM] TO CLEAR MEMORY

KEY ENTRY	CODE SHOWN	COMMENTS
RCL1	3401	N — X, B
X	71	
STO3	3303	Z
RCL2	3403	Y
gx↔y	3507	Z, Y
1	01	
+	61	Z+1, Y
STO4	3304	
÷	81	
CHS	42	
f^{-1}	32	
LN	07	
STO5	3305	$e^{-Y/(Z+1)}$
RCL1	3401	N
1	01	
gx=y	3523	1=N
RCL5	3405	
R/S	84	P1← $e^{-Y/(Z+1)}$
RCL2	3302	Y
E	15	P(N-1, Y), N-1
STO6	3306	
RCL3	3403	Z
RCL4	3404	Z+1, Z, P, N-1
÷	81	
STO4	3304	Z/(Z+1)
gR↑	3509	N-1
g	35	
y^x	05	$[Z/(Z+1)]^{N-1}$
RCL5	3405	
÷	81	
STO5	3305	S← $[Z/(Z+1)]^{N-1} e^{-Y/(Z+1)}$
EEX	43	
7	07	
gx≤y	3522	$10^7 \leq S$
RCL6	3406	
R/S	84	P1← P(N-1, Y)
CLX	44	
RCL4	3404	Z/(Z+1)
RCL2	3402	Y
X	71	YZ/(Z+1)
E	15	P[N-1, YZ/(Z+1)]
CHS	42	
1	01	
+	61	1-P
RCL5	3405	S
÷	81	(1-P)/S
RCL6	3406	P(N-1, Y)
+	61	
R/S	84	P
LBL	23	

KEY ENTRY	CODE SHOWN	COMMENTS
E	15	
STO7	3307	Y1
CHS	42	
f^{-1}	32	
LN	07	YM ← e^{-Y1}
STO8	3308	YMS ← YM
1	01	M←1
LBL	23	
D	14	
RCL1	3401	N
1	01	
-	51	N-1
gx≤y	3522	N-1 ≤ M, YM
RCL8	3408	P←YMS, N-1, M, YM
RTN	24	
RCL7	3407	Y1, N-1, M, YM
gR↑	3509	YM
X	71	YM·Y1
gR↑	3509	M
÷	81	YM←YM·Y1/M
STO	33	
+	61	
8	08	(YMS←YMS + YM)
gR↑	3509	M
1	01	
+	61	M←M+1
D	14	

REGISTERS	
R_1	N
R_2	Y
R_3	Z
R_4	Z+1 Z/(Z+1)
R_5	$e^{-Y/(Z+1)}$ S
R_6	P(N-1, Y)
R_7	Y1
R_8	YMS
R_9	Used
LABELS	
A	
B	
C	
D	YM Loop
E	Y→P
0	
1	
2	
3	
4	
5	
6	
7	
8	
9	
FLAGS	
1	
2	

TO RECORD PROGRAM INSERT MAGNETIC CARD WITH SWITCH SET AT W/PRGM

HP-65 Program Form

Title HP-65 P3 Program Listing Page ____ of ____

SWITCH TO W/PRGM PRESS [f] [PRGM] TO CLEAR MEMORY

KEY ENTRY	CODE SHOWN	COMMENTS
RCL1	3401	N X
X	71	B
2	02	
÷	81	
STO3	3303	Z/2
RCL3	3403	
1	01	
+	61	1+Z/2, Z/2
÷	81	Z/(Z+2)
RCL2	3402	Y
gLSTX	3500	1+Z/2, Y
÷	81	2Y/(Z+2), Z/(Z+2)
STO5	3305	
RCL1	3401	N
2	02	
gx≤y	3522	2≤N
GTO	22	
1	01	
gR↑	3509	
gR↑	3509	
X	71	$2YZ/(Z+2)^2$
1	01	
+	61	
RCL5	3405	2Y(Z+2)
CHS	42	
f^{-1}	32	
LN	07	-2Y/(Z+2)
X	71	e [1+2YZ/(Z+2)]^2
R/S	84	
LBL	23	
1	01	
-	51	N-2
STO6	3306	
gR↑	3509	Z/(Z+2), N-2, 2Y/(Z+2)
f	31	
LN	07	
X	71	-(N-2) ln[(Z+2)/Z]
+	61	EXP←2Y/(Z+2)-(N-2)ln[ ]
CHS	42	
f^{-1}	32	
LN	07	e^{-EXP}
1	01	
RCL6	3406	
RCL3	3403	Z/2, N-2, 1, e^{-EXP}
÷	81	2(N-2)/Z
-	51	1-2(N-2)/Z
RCL5	3405	2Y/(Z+2)
+	61	
X	71	C←[1-2(N-2)/Z+2Y/(Z+2)] e^{-EXP}
STO4	3304	

KEY ENTRY	CODE SHOWN	COMMENTS
RCL5	3405	2Y/(Z+2)
RCL3	3403	Z/2
X	71	YZ/(Z+2)
E	15	P(N-1 , YZ/(Z+2))
CHS	42	
1	01	
+	61	1-P
RCL5	3404	C
X	71	C(1-P)
STO4	3304	
RCL2	3402	P
E	15	P(N-1, Y)
gR↑	3509	YM = $Y^{N-2}e^{-Y}/(N-2)!$
RCL5	3405	2Y/(Z+2)
X	71	
+	61	P(Y) + YM · 2Y/(Z+2)
RCL4	3404	C(1-P)
+	61	
R/S	84	P3
LBL	23	
E	15	
STO7	3307	Y3
CHS	42	
f^{-1}	32	
LN	07	YM←e^{-Y3}
STO8	3308	YMS←YM
1	01	M←1
LBL	23	
D	14	
RCL1	3401	N
1	01	
-	51	N-1
gx≤y	3522	N-1≤M, YM
RCL8	3408	P←YMS, N-1, M, YM
RTN	24	
RCL7	3407	Y3, N-1, M, YM
gR↑	3509	YM
X	71	YM · Y3
gR↑	3509	M
÷	81	YM←YM · Y3/M
STO	33	
+	61	
8	08	(YMS←YMS+YM)
gR↑	3509	M
1	01	
+	61	M←M+1
D	14	
100		

REGISTERS	
R_1	N
R_2	Y
R_3	Z/2
R_4	C
	C(1-P)
R_5	2Y/(Z+2)
R_6	N-2
R_7	Y3
R_8	YMS
R_9	

LABELS	
A	
B	
C	
D	YM Loop
E	Y→P
0	
1	
2	
3	
4	
5	
6	
7	
8	
9	

FLAGS	
1	
2	

HEWLETT PACKARD

9320-0616

TO RECORD PROGRAM INSERT MAGNETIC CARD WITH SWITCH SET AT W/PRGM

HP-65 Program Form

Title HP-65 P4 Program Listing Page ___ of ___

SWITCH TO W/PRGM PRESS [f] [PRGM] TO CLEAR MEMORY

KEY ENTRY	CODE SHOWN	COMMENTS
0	00	X
STO6	3306	M←0 B
gR	3508	
2	02	
÷	81	
STO3	3303	X/2
1	01	
+	61	
STO7	3307	1+X/2
RCL1	3401	N
CHS	42	
g	35	
y^x	05	
STO4	3304	$ZK \leftarrow [2/(X+2)]^N$
RCL2	3402	Y
RCL7	3407	1+X/2
÷	81	
STO7	3307	V←2Y/(X+2)
CHS	42	
f^{-1}	32	
LN	07	$YM \leftarrow e^{-V}$
STO8	3308	YMS← YM
E	15	SUM← P(N, V)
gR↓	3508	
STO5	3305	YM
RCL4	3404	ZK
gx↔y	3522	YM, ZKS←ZK
LBL	23	
1	01	YM, ZKS
RCL7	3407	
RCL6	3406	M, V, YM, ZKS
÷	81	
X	71	
STO5	3305	YM←YM · V/M
1	01	
gR↑	3509	
-	51	1-ZKS, YM, ZKS
X	71	
STO	33	
+	61	
8	08	(SUM←SUM+YM(1-ZKS))
CLX	44	
RCL1	3401	N, ZKS
2	02	
X	71	
RCL6	3406	M, 2N, ZKS
-	51	2N-M
gLSTX	3500	M
RCL1	3401	N
-	51	M-N

KEY ENTRY	CODE SHOWN	COMMENTS
1	01	
+	61	M-N+1, 2N-M, ZKS
÷	81	[(2N-M)/(M-N+1)], ZKS
RCL4	3404	ZK
RCL3	3403	S, ZK, [ ], ZKS
X	71	
X	71	
STO4	3304	ZK←ZK · S [ ], ZKS
+	61	ZKS ← ZKS+ZK
RCL6	3406	
1	01	
+	61	
STO6	3306	M← M+1
RCL1	3401	N
2	02	
X	71	
gx≤y	3522	2N≤M
RCL8	3408	
R/S	84	P4 ← SUM
gR↑	3509	ZKS
RCL5	3405	YM, ZKS
GTO	22	
1	01	
LBL	23	
E	15	YM
RCL6	3406	
1	01	
+	61	
STO6	3306	M←M+1
RCL1	3401	
gx≤y	3522	N≤M
gR↓	3508	M, YM
RTN	24	
gR↓	3508	M, YM
÷	81	YM/M
RCL7	3407	V
X	71	YM←YM · V/M
STO	33	
+	61	
8	08	(YMS←YMS + YM)
E	15	

REGISTERS	
R_1	N
R_2	Y
R_3	X/2 =S
R_4	ZK
R_5	YM
R_6	M
R_7	1+X/2 V
R_8	YMS SUM
R_9	

LABELS	
A	
B	
C	
D	
E	YM Loop
0	
1	
2	
3	
4	
5	
6	
7	
8	
9	

FLAGS	
1	
2	

TO RECORD PROGRAM INSERT MAGNETIC CARD WITH SWITCH SET AT W/PRGM

HP-65 Program Form

Title HP-65 PG Program Listing   Page ___ of ___

SWITCH TO W PRGM   PRESS f PRGM TO CLEAR MEMORY

KEY ENTRY	CODE SHOWN	COMMENTS	
RCL1	3401	N, X, K	K
X	71		↑
STO3	3303	Z	X
gR↓	3508		B
STO4	3303	K	
RCL2	3402	Y	
CHS	42		
f^{-1}	32		
LN	07		
STO7	3307	YM←e^{-Y}	
STO8	3308	YMS←YM	
1	01		
STO6	3306	M←1	
LBL	23		
1	01		
RCL1	3401	N, M	
gx≤y	3522	N ≤ M	
GTO	22		
2	02		YM←YM· Y/M
A	11	M	YMS←YMS + YM
GTO	22		M←M+1
1	01		
LBL	23		
2	02		
RCL8	3408	YMS	
STO5	3305	SUM←YMS	
1	01		
RCL3	3403	Z	
RCL3	3403	Z	
RCL4	3404	K, Z, Z, 1	
+	61		
÷	81		
STO3	3303	V←Z/(K+Z)	
-	51	1-V	
RCL4	3404	K	
g	35		
y^x	05		
STO4	3304	XB←$(1-V)^K$	
gLSTx	3500	ZK1←K, XB, 1→	
gR↓	3508	XBS←XB, 1, 1, ZK1	
-	51	(1-XBS), 1, ZK1→	
gR↑	3509	ZK1, (1-XBS)	
LBL	23		YM←YMS· Y/M
3	03		YMS←YMS+YM
A	11	M	SUM←SUM+YM(1-XBS)
RCL1	3401	N	M←M+1
-	51	M-N	
RCL4	3404	XB	
gx↔y	3507	m-N, XB, ZK1, (1-XBS)	
÷	81	XB/(M-N)	

KEY ENTRY	CODE SHOWN	COMMENTS
RCL3	3403	V
X	71	XB · V/B
gx↔y	3507	ZK1
X	71	
STO4	3304	XB←XB · ZK1 · V/(M-N)
gLSTX	3500	ZK1
gR↓	3508	XB, (1-XBS), (1-XBS), ZK1
-	51	(1-XBS)←(1-XBS) - XB
gR↑	3509	ZK1, (1-XBS)
1	01	
+	61	ZK1←ZK1+1
1	01	
RCL8	3408	YMS, 1, ZK1, (1-XBS)
-	51	
gR↑	3509	
X	71	(1-XBS)(1-YMS), ZK1, (1-XBS)
EEX	43	
CHS	42	
8	08	10^{-8}, ( ) ( )
gx>y	3424	
RCL5	3405	P←SUM
R/S	84	
gR↑	3509	
gR↑	3509	ZK1, (1-XBS)
GTO	22	
3	03	
LBL	23	
A	11	ZK1, (1- XBS)
RCL7	3407	YM
RCL2	3402	Y, YM, ZK1, (1-XBS)
X	71	YM· Y
RCL6	3406	M, YM· Y, ZK1, (1-XBS)
÷	81	
STO7	3307	YM←YM· Y/M
STO	33	
+	61	
8	08	(YMS←YMS + YM)
gR↑	3509	(1-XBS), YM, ZK1(1-XBS)
X	71	YM(1-XBS), ZK1, ( ) →
STO	33	
+	61	
5	05	(SUM←SUM+YM(1-XBS))
gR↓	3508	ZK1, ( )
RCL6	3406	M
1	01	1, M, ZK1, (1-XBS)
+	61	
STO6	3306	M←M+1
RTN	24	

REGISTERS	
R_1	N
R_2	Y
R_3	Z, V
R_4	K, XB
R_5	SUM
R_6	M
R_7	YM
R_8	YMS
R_9	logic

LABELS	
A	Recursion
B	
C	
D	
E	
0	
1	Used
2	Used
3	Used
4	
5	
6	
7	
8	
9	

FLAGS	
1	
2	

TO RECORD PROGRAM INSERT MAGNETIC CARD WITH SWITCH SET AT W PRGM

2. FIXED-THRESHOLD, BARTON ALGORITHM

These HP-65 programs calculate the required average SNR in dB for a given detection probability and target model. The target model is specified by its probability density function, i.e., nonfluctuating, Rayleigh (ala Swerling Case I) or Rayleigh plus an constant component of equal power (ala Swerling Case III), and by its diversity order, N_e, defined as the number of independent target values within the N samples noncoherently integrated. It is always necessary to run SNRN, the calculation for a nonfluctuating target, after entering N, PF, and PD and before running SNRF, the calculation for any fluctuating target.

Specific user instructions follow:

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS		OUTPUT DATA/UNITS
1	Enter program SNRN				
2	(Repeat as desired)	N	STO	1	
		PD $\geq$ 0.3	STO	2	
		PF $\leq$ 0.5	STO	3	
			A		SNRN dB
	If desired		-		Gi dB
	To include collapsing loss or tgt fluct go to step 3				
3	Enter program SNRF				
	Samples containing signal	$1 \leq N_s \leq N$	STO	6	
	Order of target diversity	$1 \leq N_e \leq N_s$	STO	7	
	Go to step 4, 5, or 6				
4	For nonfluctuating target		A		SNRC dB
5	For Rayleigh const tgt (Inc Cases III & IV)		B		SNRF dB
6	For Rayleigh & const tgt (Inc Cases III & IV)		C		SNRF dB
	If desired		R/S		Lf dB
			R/S		Gd dB
	(Steps 4, 5, and 6 may be repeated in any order)		E		R/R_o

Gd : Diversity gain (dB)
Gi : Integration gain (dB)
Lf : Fluctuation loss (dB)
N : Signal and/or noise samples integrated
  For Cases I and III : Ne=1
  For Cases II & IV : Ne=Ns
Ns : Samples within N containing signal
PD : Probability of detection
PF : Probability of false alarm
R/Ro : Ratio of detection range to that for which SNRf = 0 dB
SNRC : SNR per sample for nonfluctuating target w/collapsing loss (dB)
SNRF : Avg SNR per sample for fluctuating target (dB)
SNRN : SNR per sample for nonfluctuating target (dB)

HP-65 Program Form

Title: HP-65 SNRN Program Listing — Page ____ of ____

SWITCH TO W/PRGM PRESS [f] [PRGM] TO CLEAR MEMORY

KEY ENTRY	CODE SHOWN	COMMENTS
0	00	
ST08	3308	
RCL2	3402	PD
2	02	
X	71	
1	01	
gx>y	3524	1 > 2PD
STO8	3308	
gR↓	3508	2PD
RCL2	3402	PD
-	51	{ PD for PD ≤ 0.5
B	12	1-PD for PD ≥ 0.5
g	35	
DSZ	83	
CHS	42	
gNOP	3501	
STO4	3304	$-Q^{-1}(PD)$
RCL3	3403	PF
B	12	$Q^{-1}(PF)$
RCL4	3404	
-	51	$[Q^{-1}(PF)+Q^{-1}(PD)]$
↑	41	
X	71	$[\ ]^2$
2	02	
÷	81	
STO4	3304	X1
RCL1	3401	N
÷	81	XN
E	15	
STO5	3305	SNRN
STO8	3308	
RCL4	3404	X1
E	15	
STO4	3304	SNR1
RCL5	3405	
R/S	84	SNRN
LBL	23	
B	12	P
↑	41	
X	71	P^2
f	31	
LN	07	$\ln P^2$
CHS	42	
f	31	
√	09	t
↑	41	
↑	41	
↑	41	
•	83	
0	00	

KEY ENTRY	CODE SHOWN	COMMENTS
4	04	
4	04	
8	08	$b_2 = 0.0448$
X	71	b_2t
•	83	
9	09	
9	09	
2	02	$b_1 = 0.992$
+	61	b_1+b_2t
X	71	$b_1t+b_2t^2$
1	01	
+	61	$1+b_1t+b_2t^2=D$
gR↑	3509	t
•	83	
2	02	
7	07	
1	01	$a_1 = 0.271$
X	71	
2	02	
•	83	
3	03	
1	01	$a_0 = 2.31$
+	61	$a_0 = a_1t = N$
gx↔y	3507	D
÷	81	N/D
-	51	$Q^{-1}(P)$
STO	33	
9	09	
RTN	24	$Q^{-1}(P)$
LBL	23	
E	15	X
↑	41	
↑	41	
↑	41	
9	09	
•	83	
2	02	
+	61	X+0.92
X	71	X(X+0.92)
f	31	
√	09	$\sqrt{X(X+0.92)}$
+	61	$x+\sqrt{\ }$
2	02	
÷	81	
f	31	
LOG	08	
1	01	
0	00	
X	71	
RTN	24	SNRX

REGISTERS

R_1	N
R_2	PD
R_3	PF
R_4	$-Q^{-1}(PD)$, X1, SNR1
R_5	SNRN
R_6	
R_7	
R_8	SNRN
R_9	Used

LABELS

A	
B	$Q^{-1}(P)$
C	
D	
E	SNRX
0	
1	
2	
3	
4	
5	
6	
7	
8	
9	

FLAGS

1	
2	

TO RECORD PROGRAM INSERT MAGNETIC CARD WITH SWITCH SET AT W/PRGM

HP-65 Program Form

Title: HP-65 SNRF Program Listing   Page ____ of ____

SWITCH TO W/PRGM PRESS [f] [PRGM] TO CLEAR MEMORY

KEY ENTRY	CODE SHOWN	COMMENTS	KEY ENTRY	CODE SHOWN	COMMENTS
LBL	23		X	71	
A	11		2	02	
0	00		-	51	D34
GTO	22		LBL	23	
1	01		2	02	D
LBL	23		f	31	
B	12		LOG	08	
RCL3	3403	PF	1	01	
f	31		0	00	
LN	07		X	71	10 log D
RCL2	3402	PD	RCL4	3404	SNR1
f	31		-	51	
LN	07		RCL7	3407	Ne
÷	81	ln PF/ln PD	÷	81	Lf
1	01		LBL	23	
-	51	D12	1	01	Lf
GTO	22		↑	41	
2	02		↑	41	
LBL	23		RCL1	3401	N
C	13		RCL6	3406	Ns
.	83		÷	81	
3	03		f	31	
6	06		LOG	08	
1	01		1	01	
RCL3	3403	PF	0	00	
f	31		X	71	10 log (N/Ns)
LOG	08		+	61	
-	51	(.361 - log PF)	RCL8	3408	SNRN
3	03		+	61	
.	83		STO5	3305	
2	02		R/S	84	SNRF
7	07		gR↓	3508	
RCL2	3402	PD	R/S	84	Lf
CHS	42		RCL7	3407	Ne
1	01		1	01	
+	61		-	51	Ne-1
·	31		X	71	
√	09		R/S	84	Gd
÷	81	$3.27/\sqrt{1-PD}$	LBL	23	
gLSTX	3500	$\sqrt{1-PD}$	E	15	
.	83		RCL5	3405	SNRF
9	09		CHS	42	
6	06		4	04	
X	71		0	00	
-	51	$(3.27/\sqrt{1-PD}) - 0.96\sqrt{1-PD}$	÷	81	
1	01		f^{-1}	32	
.	83		LOG	08	
2	02		R/S	84	R/Ro
9	09				
-	51	( )			

REGISTERS	
R_1	N
R_2	PD
R_3	PF
R_4	SNR1
R_5	SNRN SNRF
R_6	Ns
R_7	Ne
R_8	SNRN
R_9	

LABELS	
A	Nonfluct
B	Sw 1&2
C	Sw 3&4
D	
E	
0	
1	Used
2	Used
3	
4	
5	
6	
7	
8	
9	

FLAGS	
1	
2	

HEWLETT PACKARD

TO RECORD PROGRAM INSERT MAGNETIC CARD WITH SWITCH SET AT W/PRGM

3. CFAR DETECTION, RECURSIVE SOLUTIONS

These HP-65 programs calculate the detection probability, given the average sample SNR and target model for an adaptive detector threshold which is set proportional to the noncoherent integration of R noise samples. These programs require initial calculation of the threshold proportionality constant, T, or equivalently A = 1/(T+1). Two cards must be entered for the P2 case or three cards must be entered for the general case. The partitioning is such that the first program calculates T0, a starting value of an iterative solution for T. The HP-65 P2C program does the iteration for T and also calculates P2. By rerunning HP-65 T0 with a given P2, the required average SNR can also be calculated by HP-65 P2C.

The programs, PGC(1) and PGC(2), are used with T0 to calculate detection probability for the general chi-squared target model.

Specific user instructions follow:

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS		OUTPUT DATA/UNITS
1	Enter program T0	N	↑		
		R	↑		
		PF	A		T0
	Go to step 2 or 6				
2	Enter program P2C		R/S		T
	Go to step 3 or 4				
3	For P2	SNR dB	B		P2
	Repeat for new SNR or go to step 4 or 6 as desired				
4	For SNR, given P2, enter program T0	P2	C		T20
5	Enter program P2C		R/S		SNR dB
	Repeat steps 4 & 5 for new P2 or to to step 3 or 6 as desired				
6	For general target model enter program PGC(1)		R/S		A
7		SNR dB	↑		
		K	A		1
8	Enter program PGC(2)		R/S		P
	For new SNR or K enter PGC(1) and go to step 7				

A : 1/(T+1)
K : Chi-squared distribution parameter
N : Signal and/or noise samples integrated
PF : Probability of false alarm
P : Probability of detection for chi-squared target
P0 : Probability of detection for nonfluctuating target
P1-P4 . Probability of detection for Swerling Cases I-IV
R : Noise samples integrated to set threshold
SNR : 10 log X
T : Threshold setting divisor
T0 : Iterative solution start value for T
T20 : Iterative solution start value for T2=T/(1+X)
X : Average sample S/N within N samples

Case 0 : K-10^5 (3 place acc)
Case I : K=1
Case II : K=N
(Genl Rayleigh target : K=Ne)
Case III : K=2
Case IV : K=2N
(Genl Rayleigh + equal constant target : K=2Ne)
Weinstock : 0 < K < 1

HP-65 Program Form

Title HP-65 T0 Program Listing Page ___ of ___

SWITCH TO W/PRGM PRESS f PRGM TO CLEAR MEMORY

KEY ENTRY	CODE SHOWN	COMMENTS
LBL		← P2
C		C
f		
SF1		
STO4		P2
RCL3		TF SAVE TF
STO8		IN R5
GTO		
1		
LBL		N
A		← ↑
STO4		PF, R, N R
g↓		↑
STO2		R PF
g↓		A
STO1		N
LBL		
1		
RCL4		PIN
f		
LOG		
CHS		L
RCL1		N
↑		
f		
√		$\sqrt{N}$
-		$N-\sqrt{N}$, L
gLSTx		$\sqrt{N}$
1		
-		$\sqrt{N}-1$, $N-\sqrt{N}$, L→
gR↑		L
f		
√		$\sqrt{L}$
+		$\sqrt{L}+\sqrt{N}-1$
gLSTx		$\sqrt{L}$
÷		$(\sqrt{L}+\sqrt{N}-1/\sqrt{L}$, $N-\sqrt{N}$, L→
gx↔y		
gR↑		L, $N-\sqrt{N}$
÷		
2		
.		
3		
÷		$(N-\sqrt{N})/2.3L$, $(\sqrt{L}+\sqrt{N}-1)/\sqrt{L}$
+		
RCL2		R
1		
-		R-1
RCL2		R
.		
9		

KEY ENTRY	CODE SHOWN	COMMENTS
2		
2		
+		R+.922, R-1
÷		B
X		B( )
1		
gLSTx		B, 1, B( )
-		
RCL1		N
X		(1-B) N, B( )
+		[ ]
1		
RCL4		PIN
RCL2		R
g		
1/X		
g		
y^x		$PIN^{1/R}$, 1, [ ]
-		
gLSTx		
÷		$(1-PIN^{1/R})/PIN^{1/R}$, [ ]
X		1/T0
g		
1/X		
STO3		T0
R/S		

REGISTERS	
R_1	N
R_2	R
R_3	T0
R_4	PIN
R_5	
R_6	
R_7	
R_8	TF
R_9	
LABELS	
A	PIN ← PF
B	
C	PIN ← P2
D	
E	
0	
1	
2	
3	
4	
5	
6	
7	
8	
9	
FLAGS	
1	PIN ← P2
2	

TO RECORD PROGRAM INSERT MAGNETIC CARD WITH SWITCH SET AT W/PRGM

HP-65 Program Form

Title HP-65 P2C Program Listing Page ___ of ___

SWITCH TO W/PRGM PRESS f PRGM TO CLEAR MEMORY

KEY ENTRY	CODE SHOWN	COMMENTS
RCL3		T0
D		P
RCL4		PIN
÷		P/PIN
f		
LN		ln (P/PIN)
RCL6		Q
RCL5		P
÷		Q/P
RCL2		R
RCL3		T
÷		R/T, Q/P, ln(P/PIN)
-		
÷		ΔT
STO		
+		
3		(T←T+ΔT)
RCL3		T
÷		
g		
ABS		
EEX		
CHS		
6		10^{-6}, |ΔT/T|
gx≤y		
GTO		
0		RETURN TO PRGM START
RCL3		T
f^{-1}		
TF1		
R/S		T
gNOP		
f^{-1}		
SF1		
RCL8		T, T2 } RESTORE T
STO3		IN R3
÷		
1		
-		X
f		
LOG		
1		
0		
X		
R/S		SNR dB
LBL		
B		SNR dB
1		
0		
÷		

KEY ENTRY	CODE SHOWN	COMMENTS
f^{-1}		
LOG		
1		
+		
RCL3		T
X		T2←T (1+X)
D		
R/S		P2
LBL		
D		T
↑		
↑		
1		
+		
STO7		T+1
÷		
RCL2		R
g		
y^x		$PM \leftarrow [T/(T+1)]^R$
STO5		PMS←PM
0		M←0
STO6		QMS←0
LBL		
1		M, PM
RCL2		R
gx↔y		M, R, PM
+		R+M
gLSTx		M
g↓		R+M, PM, ~M
X		
RCL7		T+1
÷		M· PM←PM(R+M)/(T+1)
STO		
+		
6		(QMS←QMS+M· PM)
gR↑		M
1		
+		M←M+1
RCL1		N, M, M· PM
gx≤y		
RCL5		
RTN		P←PMS
gR↓		M, M· PM
÷		PM
STO		
+		
5		(PMS←PMS+PM)
gLSTx		M, PM
GTO		
1		

REGISTERS	
R_1	N
R_2	R
R_3	T
R_4	PIN
R_5	PMS
R_6	QMS
R_7	T+1
R_8	T
R_9	logic

LABELS	
A	
B	SNR→P2
C	
D	T→P
E	
0	PRGM ST
1	
2	
3	
4	
5	
6	
7	
8	
9	

FLAGS	
1	P2←PIN
2	

HEWLETT PACKARD

TO RECORD PROGRAM INSERT MAGNETIC CARD WITH SWITCH SET AT W/PRGM

HP-65 Program Form

Title HP-65 PGC(1) Program Listing Page ____ of ____

SWITCH TO W/PRGM PRESS f PRGM TO CLEAR MEMORY

KEY ENTRY	CODE SHOWN	COMMENTS	KEY ENTRY	CODE SHOWN	COMMENTS
LBL			RCL5		P← PMS
0			÷		Q/P
RCL3			RCL2		R
RCL3			RCL3		T
1		1, T, T	÷		R/T, Q/P, ln (P/PIN)
+			−		
STO7		T+1	÷		ΔT
÷			STO		
RCL2		R	+		
g			3		(T← T+ΔT)
y^x		PM← [T/(T+1)]^R	RCL3		T
STO5		PMS← PM	÷		
0		M← 0	g		
STO6		QMS← 0	ABS		|ΔT/T|
LBL			EEx		
1		M, PM	CHS		
RCL2		R	6		10^{-6}, |ΔT/T|
gx↔y		M, R, PM	gx≤y		
+		R+M	GTO		
gLSTx		M	0		
gR↓		R+M, PM, ~, M	RCL3		T
X			1		
RCL7		T+1	+		
÷		M·PM	g		
STO			1/X		
+			STO3		A← 1/(T+1)
6		(QMS← QMS+M·PM)	R/S		SNR
gR↑		M	LBL		↑
1			A		K
+		M← M+1	STO5		K A
RCL1		N, M	gR↓		SNR
gx≤y			1		
GTO			0		
2			÷		
gR↓		M, M·PM	f^{-1}		
÷		PM	LOG		X
STO			RCL1		N
+			X		
5		(PMS←PMS+PM)	STO4		Z
gLSTx		M, PM	1		
GTO			RCL3		A
1			−		
LBL			RCL2		R
2			g		
RCL5		P← PMS	y^x		
RCL4		PIN	STO7		PM←(1-A)^R
÷			STO8		PMS← PM
f			1		
LN		ln (P/PIN)	STO6		M← 1
RCL6		Q← QMS	R/S		

REGISTERS	
R_1	N
R_2	R
R_3	T, A
R_4	PIN, Z
R_5	PMS, K
R_6	QMS, M
R_7	T+1, PM
R_8	PMS
R_9	logic

LABELS	
A	SNR, K entry
B	
C	
D	
E	
0	
1	
2	
3	
4	
5	
6	
7	
8	
9	

FLAGS	
1	
2	

TO RECORD PROGRAM INSERT MAGNETIC CARD WITH SWITCH SET AT W/PRGM

HP-65 Program Form

Title HP-65 PGC(2) Program Listing Page ___ of ___

SWITCH TO W/PRGM PRESS [f] [PRGM] TO CLEAR MEMORY

KEY ENTRY	CODE SHOWN	COMMENTS
LBL		
1		1
RCL1		N
gx≤y		
GTO		
2		
A		
GTO		
1		
LBL		
2		
1		
RCL4		
RCL4		
RCL5		K, Z, Z, 1
+		
÷		
STO4		V←Z/(K+Z), 1→
-		
RCL5		K
g		
y^x		
STO5		XB←$[K/(K+Z)]^K$
gLSTx		K
1		
-		K-1
RCL8		SUM, K-1, XB, 1
gR↓		
gR↓		XB, 1, SUM, K-1
-		
STO		
9		(1-XBS)←1-XB
gR↑		
LBL		
3		~, ~, SUM, K-1
+		g
CLX		0, SUM, K-1 g
A		M, SUM, K-1
CLX		
RCL		
9		1-XBS, SUM, K-1
RCL7		PM, 1-XBS, SUM, K-1
X		
+		SUM←SUM+PM(1-XBS)
RCL6		M
RCL1		N, M, SUM, K-1
-		M-N
gR↑		K-1
gx↔y		M-N, K-1
÷		

KEY ENTRY	CODE SHOWN	COMMENTS
gLSTx		
÷		(K+M-N-1)/(M-N), SUM, K-1→
RCL4		V
X		
RCL5		XB
X		
STO5		XB←XB·V(K+M-N-1)/(M-N)
STO		
-		
9		{(1-XBS)←(1-XBS)-XB}
CLX		
1		
RCL8		PMS, 1, SUM, K-1
-		1-PMS
EEX		
8		
X		
g		
1/X		
RCL		
9		1-XBS, $1/10^8$(1-PMS), SUM, K-1
gx>y		
GTO		
3		
gR↓		
gR↓		
R/S		P←SUM
LBL		
A		SUM, K-1
RCL2		R
RCL6		M, R, SUM, K-1
+		
1		
-		
RCL6		M
÷		(R+M-1)/M, SUM, K-1
RCL3		A
X		
RCL7		PM
X		
STO7		PM←PM·A(R+M-1)/M
STO		
+		
8		(PMS←PMS+PM)
gR↓		SUM, K-1
RCL6		M
1		
+		
STO6		M←M+1
RTN		M, SUM, K-1

REGISTERS

R_1	N
R_2	R
R_3	A
R_4	Z, V
R_5	K, XB
R_6	M
R_7	PM
R_8	PMS
R_9	logic, 1-XBS

LABELS

A	PM, PMS, AM Recurs.
B	
C	
D	
E	
0	
1	Initial
2	PM loop
3	Sum loop
4	
5	
6	
7	
8	
9	

FLAGS

1	
2	

HEWLETT hp PACKARD

SECTION VI

HP-67 PROGRAMS

The HP-67 is very similar to its predecessor, the HP-65. As a result, programs written for the HP-65, such as theprograms of this report, can almost be transcribed one-to-one for the HP-67. The major difference is the greater memory and programming capacity of the 67. This was used to combine and store multiple, related, HP-65 programs on single HP-67 program cards.

The programming differences between the two calculators that prevent exact one-to-one transcription are noted here for future reference:

- MERGED INSTRUCTIONS

  The HP-67 has more merged instructions; e.g., "STO + 8" on the HP-67 requires 3 program lines on the 65.

- CONDITIONAL BRANCHING

  The HP-65 skips over two program steps if the conditional test is false. The HP-67 only skips one program step when the test is false.

- INDEX REGISTER

  The HP-65 uses register R8 as an index register. The index register in the HP-67 is denoted "I". It can be used for real number storage as well. The register R8 in the HP-67 is for data storage only.

- PROGRAM STORAGE

  The HP-65 has a capacity of 100 program steps. The HP-67 has 224.

- DATA STORAGE

  The HP-65 has 9 storage registers including register 9 which is not fully available because it is used for internal subroutines. The HP-67 has 26 data storage registers including the "I" register. All are fully available.

- LABELS

  The HP-65 has 15 labels for program entry points, subroutines, and branch points. The HP-67 has 20.

The HP-67 detection programs are not as completely annotated as the HP-65 programs. However, the HP-67 programs can be readily related to the corresponding HP-65 programs. Program steps and labels that are different are noted.

1. FIXED-THRESHOLD, RECURSIVE PROGRAMS

These HP-67 programs calculate the detection probability, given the number of samples noncoherently integrated, the false-alarm probability, and the average sample signal-to-noise ratio (SNR) (in dB) for the various target models. Unlike the HP-65 programs, each program includes the threshold determination. The P1 and P2 calculations are also included on a single program. In addition, the P1-P2 program can also calculate by iteration the value of SNR required for a Case II target with a given P2.

Specific user instructions follow:

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS		OUTPUT DATA/UNITS
1.	Enter P0, P1-P2, P3, P4 or PG program	N			
		PF	A		PF (calc)
	Go to step 2, 3, 4 or 5 as appropriate				
2.	For P0, P2, P3 or P4 do either:	SNR dB	B		P
	or:	X	f	b	P
	Repeat as desired				
	For P2 go to step 3 if desired				
3.	For SNR or X given P2 (P2 prgm)	P2	C		SNR dB
			RCL	A	X
	Repeat or go to step 2 as desired				
4.	For P1 do either:	SNR dB	D		P1
	or:	X	f	d	P1
	Repeat as desired				
5.	For P6 do either:	SNR dB	B		K
	or:	X	f	b	K
6.	Enter desired K if different from display	K	C		P
	Repeat or go to step 5 as desired				
	After steps 2, 3, 4 or 6 do any of the following if desired:		RCL	0	PF (calc)
			RCL	1	N
	(May be done after step 1)		RCL	2	Y
			RCL	9	P
			RCL	A	X
			RCL	B	SNR dB
	(PG program only)		RCL	E	K

K : Chi-squared distribution parameter
N : Signal and/or noise samples integrated
Ne : Target diversity within N samples
PF : Probability of false alarm
P : Probability of detection for chi-squared target
P0 : Probability of detection for nonfluctuating target
P1-P4 : Probability of detection for Cases I-IV
SNR : 10 log X
X : Average sample power S/N within N
Y : Fixed detection threshold
Y2 : Y/(1+X)

Case 0 : $K=10^5$ (~3 place acc)
Case I : K=1
Case II : K=N
(Genl Rayleigh target : K=Ne)
Case III : K=2
Case IV : K=2N
(Genl Rayleigh + equal constant target : K=Ne)
Weinstock : $0 < K < 1$

HP-67 P0 Program Listing

STEP	KEY ENTRY	KEY CODE	COMMENTS
001	LBL A	312511	
	STO4	3304	
	LOG	3153	
	CHS	42	
	X↔Y	3552	
	STO1	3301	
	↑	41	
	√	3154	
	-	51	
010	LSTX	3582	
	1	01	
	-	51	
	R^	3554	
	√	3154	
	+	61	
	LSTX	3582	
	X	71	
	2	02	
	.	83	
020	3	03	
	X	71	
	+	61	
	STO2	3302	
	LBL1	312501	
	GSBE	312215	
	RCL8	3408	
	R^	3554	
	÷	81	
	RCL8	3408	
030	RCL4	3404	
	÷	81	
	Ln	3152	
	X	71	
	STO+2	336102	
	LSTX	3582	
	ABS	3564	
	EEX	43	
	CHS	42	
	6	06	
040	X≤Y?	3271	
	GTO1	2201	
	RCL8	3408	
	STO0	3300	
	RTN	3522	
	LBLE	312515	
	1	01	
	2	02	
	CHS	42	
	STI	3533	
050	RCL2	3402	
	CHS	42	
	e^x	3252	
	STO8	3308	
	0	00	
	↑	41	
	1	01	
	+	61	
	RCL1	3401	
	X≤Y?	3271	
060	RTN	3522	
	RCL2	3402	
	R^	3554	
	X	71	
	R^	3554	
	÷	81	
	STO+8	336108	
	R^	3554	
	GTO(i)	2224	
	R/S	84	
070	LBL B	312512	
	1	01	
	0	00	
	÷	81	
	10^x	3253	
	LBL b	322512	
	STO A	3311	
	LOG	3153	
	1	01	
	0	00	
080	X	71	
	STO B	3312	
	RCL A	3411	
	RCL1	3401	
	X	71	
	STO3	3303	
	RCL2	3402	
	CHS	42	
	e^x	3252	
	STO7	3307	
090	STO8	3308	
	1	01	
	STO6	3306	
	LBL 0	312500	
	RCL1	3401	
	X≤Y?	3271	
	GTO2	2202	
	D	312214	
	1	01	
	RCL6	3406	
100	+	61	
	STO6	3306	
	GTO0	2200	
	LBL2	312502	
	RCL8	3408	
	STO5	3305	
	RCL3	3403	
	CHS	42	
	e^x	3252	
	STO4	3304	
110	R∨	3553	
	LBL3	312503	
	R^	3554	

REGISTERS

0	1	2	3	4	5	6	7	8	9
PF_C	N	Y	Z	PF/X_K	SUM	M	YM	YMS	P0
S0	S1	S2	S3	S4	S5	S6	S7	S8	S9

A	B	C	D	E	I
X	SNR dB				

HP-67 P0 Program Listing (Cont)

STEP	KEY ENTRY	KEY CODE	COMMENTS
	D	312214	
	R ∨	3553	
	CHS	42	
	1	01	
	+	61	
	RCL7	3407	
	X	71	
120	STO+5	336105	
	R ∧	3554	
	RCL4	3404	
	RCL3	3403	
	X	71	
	RCL6	3406	
	1	01	
	+	61	
	STO6	3306	
	RCL1	3401	
130	−	51	
	÷	81	
	STO4	3304	
	+	61	
	↑	41	
	CHS	42	
	↑	41	
	1	01	
	+	61	
	RCL8	3408	
140	CHS	42	
	1	01	
	+	61	
	X	71	
	EEX	43	
	CHS	42	
	9	09	
	X ≤ Y ?	3271	
	GTO3	2203	
	RCL5	3405	
150	STO9	3309	
	RTN	3522	
	LBL D	312514	
	RCL7	3407	
	RCL2	3402	
	RCL6	3406	
	÷	81	
	X	71	
	STO7	3307	
	STO+8	336108	
160	RTN	3522	

STEP	KEY ENTRY	KEY CODE	COMMENTS
170			
180			
190			
200			
210			
220			

LABELS					FLAGS
A N↑ PF→	B SNR dB→	C	D "A" SUB	E Y SUB	0
a	b X→	c	d	e	1
0 Loop	1 Y Loop	2 Branch	3 P0 Loop	4	2
5	6	7	8	9	3

SET STATUS

FLAGS ON OFF	TRIG	DISP
0 ☐ ☐	DEG ☐	FIX ☐
1 ☐ ☐	GRAD ☐	SCI ☐
2 ☐ ☐	RAD ☐	ENG ☐
3 ☐ ☐		n____

HP-67 P1-P2 Program Listing

STEP	KEY ENTRY	KEY CODE	COMMENTS
001	LBL A	312511	
	STO4	3304	
	LOG	3153	
	CHS	42	
	X↔Y	3552	
	STO1	3301	
	↑	41	
	$\sqrt{\ }$	3154	
	−	51	
010	LSTX	3582	
	1	01	
	−	51	
	R^	3554	
	$\sqrt{\ }$	3154	
	+	61	
	LSTX	3582	
	X	71	
	2	02	
	.	.	
020	3	03	
	X	71	
	+	61	
	STO2	3302	
	LBL1	312501	
	E	312215	
	RCL8	3408	
	R^	3554	
	÷	81	
	RCL8	3408	
030	RCL4	3404	
	÷	81	
	Ln	3152	
	X	71	
	STO+2	336102	
	LSTX	3582	
	ABS	3564	
	EEX	43	
	CHS	42	
	6	06	
040	X ≤ Y ?	3271	
	GTO1	2201	
	RCL8	3408	
	F ? 2	357102	
	GTO0	2200	
	STO0	3300	
	RTN	3522	
	LBL 0	312500	
	STO9	3309	
	RTN	3522	
050	LBL E	312515	
	1	01	
	2	02	
	CHS	42	
	STI	3533	
	RCL2	3402	
	CHS	42	

STEP	KEY ENTRY	KEY CODE	COMMENTS
	e^x	3252	
	STO8	3308	
	0	00	
060	↑	41	
	1	01	
	+	61	
	RCL1	3401	
	X ≤ Y ?	3271	
	RTN	3522	
	RCL2	3402	
	R^	3554	
	X	71	
	R^	3554	
070	÷	81	
	STO+8	336108	
	R^	3554	
	GTO(6)	2224	
	R/S	84	
	LBL B	312512	
	1	01	
	0	00	
	÷	81	
	10^x	3253	
080	LBL b	322512	
	STOA	3311	
	LOG	3153	
	1	01	
	0	00	
	X	71	
	STOB	3312	
	RCLA	3411	
	1	01	
	X	61	
090	RCL2	3402	
	STO 6	3306	
	X↔Y	3552	
	÷	81	
	STO2	3302	
	E	312215	
	RCL6	3406	
	STO2	3302	
	RCL8	3408	
	STO9	3309	
100	RTN	3522	
	LBL C	312513	
	SF2	355102	
	RCL2	3402	
	STO6	3306	
	R v	3553	
	RCL1	3401	
	X↔Y	3552	
	A	312211	
	RCL2	3402	
110	RCL6	3406	
	STO2	3302	
	X↔Y	3552	

REGISTERS

0	1	2	3	4	5	6	7	8	9
PF_C	N	Y	Z	PF		YTEMP	Y1	YMS	P

S0	S1	S2	S3	S4	S5	S6	S7	S8	S9

A	B	C	D	E	I
X	SNR dB				

HP-67 P1-P2 Program Listing (Cont)

STEP	KEY ENTRY	KEY CODE	COMMENTS	STEP	KEY ENTRY	KEY CODE	COMMENTS
	STO6	3306			Y^x	3563	
	÷	81		170	RCL5	3405	
	1	01			÷	81	
	–	51			STO5	3305	
	STOA	3311			EEX	43	
	LOG	3153			7	07	
	1	01			X≤Y ?	3271	
120	0	00			GTO7	2207	
	X	71			CLX	44	
	STOB	3312			RCL4	3404	
	RTN	3522			RCL2	3402	
	LBLD	312514		180	E	312215	
	1	01			X	71	
	0	00			GSB8	312208	
	÷	81			CHS	42	
	10^x	3253			1	01	
	LBLd	322514			+	61	
130	STOA	3311			RCL5	3405	
	LOG	3153			÷	81	
	1	01			RCL6	3406	
	0	00			+	61	
	X	71		190	STO9	3309	
	STOB	3312			RTN	3522	
	1	01			LBL7	312507	
	4	04			RCL6	3406	
	CHS	42			RTN	3522	
	STI	3533			LBL8	312508	
140	RCLA	3411			STO7	3307	
	RCL1	3401			CHS	42	
	X	71			e^x	3252	
	STO3	3303			STO8	3308	
	RCL2	3402		200	1	01	
	X↔Y	3552			RCL1	3401	
	1	01			1	01	
	+	61			–	51	
	STO4	3304			X ≤ Y ?	3271	
	÷	81			GTO6	2206	
150	CHS	42			RCL7	3407	
	e^x	3252			R ^	3554	
	STO5	3305			X	71	
	1	01			R ^	3554	
	RCL1	3401		210	÷	81	
	X > Y ?	3281			STO + 8	336108	
	GTO9	2209			R ^	3554	
	RCL5	3405			1	01	
	STO9	3309			+	61	
	RTN	3522			GTO (i)	2224	
160	LBL9	312509			LBL6	312506	
	RCL2	3402			RCL8	3408	
	GSB8	312208			RTN	3522	
	STO6	3306					
	RCL3	3403		220			
	RCL4	3404					
	÷	81					
	STO4	3304					
	R ^	3554					

LABELS					FLAGS
A N↑PF→	B SNR dB→P2	C P2→X	D SNR dB→	E Y SUB	0
a	b X→P2	c	d X→	e	1
0	1 Y LOOP	2	3	4	2
5	6 BRANCH	7 BRANCH	8 SUB	9 BRANCH	3

SET STATUS

FLAGS	ON	OFF	TRIG		DISP	
0	☐	☐	DEG	☐	FIX	☐
1	☐	☐	GRAD	☐	SCI	☐
2	☐	☐	RAD	☐	ENG	☐
3	☐	☐			n ___	

HP-67 P3 Program Listing

STEP	KEY ENTRY	KEY CODE	COMMENTS	STEP	KEY ENTRY	KEY CODE	COMMENTS
001	LBLA	312511			+	61	
	STO4	3304			RCL1	3401	
	LOG	3153			X ≤ Y	3271	
	CHS	42		060	RTN	3522	
	X↔Y	3552			RCL2	3402	
	STO1	3301			R ∧	3554	
	↑	41			X	71	
	$\sqrt{\ }$	3154			R ∧	3554	
	−	51			÷	81	
010	LSTX	3582			STO + 8	336108	
	1	01			R ∧	3554	
	−	51			GTO(i)	2224	
	R ∧	3554			LBLB	312512	
	$\sqrt{\ }$	3154		070	1	01	
	+	61			0	00	
	LSTX	3582			÷	81	
	X	71			10^x	3253	
	2	02			LBLb	322512	
	.	83			STOA	3311	
020	3	03			LOG	3153	
	X	71			1	01	
	+	61			0	00	
	STO2	3302			X	71	
	LBL1	312501		080	STOB	3312	
	GSBE	312215			RCLA	3411	
	RCL8	3408			RCL1	3401	
	R ∧	3554			X	71	
	÷	81			2	02	
	RCL8	3408			÷	81	
030	RCL4	3404			STO3	3303	
	÷	81			RCL3	3403	
	Ln	3152			1	01	
	X	71			+	61	
	STO + 2	336102		090	÷	81	
	LSTX	3582			RCL2	3402	
	ABS	3564			LSTX	3582	
	EEX	43			÷	81	
	CHS	42			STO5	3305	
	6	06			RCL1	3401	
040	X ≤ Y	3271			2	02	
	GTO1	2201			X≤Y ?	3271	
	RCL8	3408			GTO0	2200	
	STO0	3300			R ∧	3554	
	RTN	3522		100	R ∧	3554	
	LBLE	312515			X	71	
	1	01			1	01	
	2	02			+	61	
	CHS	42			RCL5	3405	
	STI	3533			CHS	42	
050	RCL2	3402			e^x	3252	
	CHS	42			X	71	
	e^x	3252			R/S	84	
	STO8	3308			LBL0	312500	
	0	00		110	−	51	
	↑	41			STO6	3306	
	1	01			R ∧	3554	

REGISTERS

0	1	2	3	4	5	6	7	8	9
PF_C	N	Y	Z/2	PF	2Y / Z+2	N−2	Y3	YMS	P
S0	S1	S2	S3	S4	S5	S6	S7	S8	S9

A	B	C	D	E	I

JP-67 P3 Program Listing (Cont)

STEP	KEY ENTRY	KEY CODE	COMMENTS
	LN	3152	
	X	71	
	+	61	
	CHS	42	
	e^x	3252	
	1	01	
	RCL6	3406	
120	RCL3	3403	
	÷	81	
	−	51	
	RCL5	3405	
	+	61	
	X	71	
	STO4	3304	
	RCL5	3405	
	RCL3	3403	
	X	71	
130	GSB2	312202	
	RCL8	3408	
	CHS	42	
	1	01	
	+	61	
	RCL4	3404	
	X	71	
	STO4	3304	
	RCL2	3402	
	GSB2	312202	
140	RCL8	3408	
	R^	3554	
	RCL5	3405	
	X	71	
	+	61	
	RCL4	3404	
	+	61	
	STO9	3309	
	R/S	84	
	LBL2	312502	
150	STO7	3307	
	CHS	42	
	e^x	3252	
	STO8	3308	
	1	01	
	LBLD	312514	
	RCL1	3401	
	1	01	
	−	51	
	X≤Y?	3271	
160	RCL8	3408	
	RTN	3522	
	RCL7	3407	
	R^	3554	
	X	71	
	R^	3554	
	÷	81	
	STO+8	336108	
	R^	3554	

STEP	KEY ENTRY	KEY CODE	COMMENTS
	1	01	
170	+	61	
	GTOD	312214	
	R/S	84	
180			
190			
200			
210			
220			

LABELS

A	B	C	D	E
N↑ PF→	SNR dB→		P3 LOOP	Y SUB
a	b	c	d	e
0 P3 LOOP	1 Y LOOP	2 P3 SUB	3	4
5	6	7	8	9

FLAGS: 0, 1, 2, 3

SET STATUS

FLAGS (ON / OFF)	TRIG	DISP
0 ☐ ☐	DEG ☐	FIX ☐
1 ☐ ☐	GRAD ☐	SCI ☐
2 ☐ ☐	RAD ☐	ENG ☐
3 ☐ ☐		n____

HP-67 P4 Program Listing

STEP	KEY ENTRY	KEY CODE	COMMENTS
001	LBL A	312511	
	STD4	3304	
	LOG	3153	
	CHS	42	
	X⟷Y	3552	
	STO1	3301	
	↑	41	
	$\sqrt{\ }$	3154	
	−	51	
010	LSTX	3582	
	1	01	
	−	51	
	R∧	3554	
	$\sqrt{\ }$	3154	
	+	61	
	LSTX	3582	
	X	71	
	2	02	
	.	83	
020	3	03	
	X	71	
	+	61	
	STO2	3302	
	LBL1	312501	
	E	312215	
	RCL8	3408	
	R∧	3554	
	÷	81	
	RCL8	3408	
030	RCL4	3404	
	÷	81	
	LN	3152	
	X	71	
	STO+2	336102	
	LSTX	3582	
	ABS	3564	
	EEX	43	
	CHS	42	
	6	06	
040	X≤Y ?	3271	
	GTO1	2201	
	RCL8	3408	
	STO0	3300	
	RTN	3522	
	LBL E	312515	
	1	01	
	2	02	
	CHS	42	
	STI	35[illegible]3	
050	RCI	[illegible]	
	CHS	42	
	e^x	32[illegible]	
	STO8	3308	
	0	00	
	↑	41	
	1	01	
	+	61	
	RCL1	3401	
	X≤Y ?	3271	
060	RTN	3522	
	RCL2	3402	
	R∧	3554	
	X	71	
	R∧	3554	
	÷	81	
	STO+8	336108	
	R∧	3554	
	GTO (i)	2224	
	R/S	84	
070	LBL B	312512	
	1	01	
	0	00	
	÷	81	
	10^x	3253	
	LBL b	322512	
	STOA	3311	
	LOG	3153	
	1	01	
	0	00	
080	X	71	
	STOB	3312	
	RCLA	3411	
	0	00	
	STO6	3306	
	R∨	3553	
	2	02	
	÷	81	
	STO3	3303	
	1	01	
090	+	61	
	STO7	3307	
	RCL1	3401	
	CHS	42	
	Y^x	3563	
	STO4	3304	
	RCL2	3402	
	RCL7	3407	
	÷	81	
	STO7	3307	
100	CHS	42	
	e^x	3252	
	STO8	3308	
	GSB 2	312202	
	R∨	3553	
	R∨	3553	
	STO5	3305	
	RCL4	3404	
	X⟷Y	3552	
	LBL3	312503	
110	RCL7	3407	
	RCL6	3406	
	÷	81	

REGISTERS

0	1	2	3	4	5	6	7	8	9
PF_C	N	Y	X/2	$PF/_{ZK}$	YM	M	1+X/2 / V	YMS	P
S0	S1	S2	S3	S4	S5	S6	S7	S8	S9

A	B	C	D	E	I
X	SNR dB				

HP-67 P4 Program Listing (Cont)

STEP	KEY ENTRY	KEY CODE	COMMENTS	STEP	KEY ENTRY	KEY CODE	COMMENTS
	X	71					
	STO5	3305		170			
	1	01					
	R ^	3554					
	-	51					
	X	71					
	STO + 8	336108					
120	CLX	44					
	RCL1	3401					
	2	02					
	X	71					
	RCL	3406		180			
	-	51					
	LSTX	3582					
	RCL1	3401					
	-	51					
	1	01					
130	+	61					
	÷	81					
	RCL4	3404					
	RCL3	3403					
	X	71		190			
	X	71					
	STO4	3304					
	+	61					
	RCL6	3406					
	1	01					
140	+	61					
	STO6	3306					
	RCL1	3401					
	2	02					
	X	71		200			
	X ≤ Y ?	3271					
	GTO4	2204					
	R ^	3554					
	RCL5	3405					
	GTO3	2203					
150	LBL4	312504					
	RCL8	3408					
	STO9	3309					
	RTN	3522					
	LBL2	312502		210			
	RCL6	3406					
	1	01					
	+	61					
	STO6	3306					
	RCL1	3401					
160	X ≤ Y ?	3271					
	RTN	3522					
	R v	3553					
	÷	81					
	RCL7	3407		220			
	X	71					
	STO + 8	336108					
	GTO2	2202					
	R/S	84					

LABELS

A N↑ PF→	B SNR dB→P	C	D	E Y SUB
a	b X→P	c	d	e
0	1	2 YM LOOP	3 LOOP	4 BRANCH
5	6	7	8	9

FLAGS

0
1
2
3

SET STATUS

FLAGS	ON	OFF	TRIG		DISP	
0	☐	☐	DEG	☐	FIX	☐
1	☐	☐	GRAD	☐	SCI	☐
2	☐	☐	RAD	☐	ENG	☐
3	☐	☐			n ____	

HP-67 PG Program Listing

STEP	KEY ENTRY	KEY CODE	COMMENTS
001	LBL A	312511	
	STO4	3304	
	LOG	3153	
	CHS	42	
	X ≤ Y	3552	
	STO1	3301	
	↑	41	
	√	3154	
	−	51	
010	LSTX	3582	
	1	01	
	−	51	
	R ∧	3554	
	√	3154	
	+	61	
	LSTX	3582	
	X	71	
	2	02	
	•	83	
020	3	03	
	X	71	
	+	61	
	STO2	3302	
	LBL1	312501	
	E	312215	
	RCL8	3408	
	R ∧	3554	
	÷	81	
	RCL8	3408	
030	RCL4	3404	
	÷	81	
	LN	3152	
	X	71	
	STO + 2	336102	
	LSTX	3582	
	ABS	3564	
	EEX	43	
	CHS	42	
	6	06	
040	X ≤ Y ?	3271	
	GTO1	2201	
	RCL8	3408	
	STO0	3300	
	RTN	3522	
	LBLE	312515	
	1	01	
	2	02	
	CHS	42	
	STI	3533	
050	RCL2	3402	
	CHS	42	
	e^x	3252	
	STO8	3308	
	0	00	
	↑	41	
	1	01	
	+	61	
	RCL1	3401	
	X ≤ Y ?	3271	
060	RTN	3522	
	RCL2	3402	
	R ∧	3554	
	X	71	
	R ∧	3554	
	÷	81	
	STO + 8	336108	
	R ∧	3554	
	GTO (i)	2224	
	R/S	84	
070	LBL B	312512	
	1	01	
	0	00	
	÷	81	
	10^x	3253	
	LBL b	322512	
	STO A	3311	
	LOG	3153	
	1	01	
	0	00	
080	X	71	
	STOB	3312	
	RCLA	3411	
	RCL1	3401	
	X	71	
	STC13	3313	
	RCLE	3415	
	R/S	84	
	LBL C	312513	
	STOE	3315	
090	RCL2	3402	
	CHS	42	
	e^x	3252	
	STO7	3307	
	STO8	3308	
	1	01	
	STO6	3306	
	LBL0	312500	
	RCL1	3401	
	X ≤ Y ?	3271	
100	GTO2	2202	
	GSB a	322211	
	GTO0	2200	
	LBL2	312502	
	RCL8	3408	
	STO5	3305	
	1	01	
	RCL13	3413	
	RCL13	3413	
	RCLE	3415	
110	+	61	
	÷	81	
	STO3	3303	

REGISTERS

0	1	2	3	4	5	6	7	8	9
PF_C	N	Y	V	PF / XB	SUM	M	YM	YMS	P
S0	S1	S2	S3	S4	S5	S6	S7	S8	S9

A	B	C	D	E	I
X	SNR dB	Z		K	

HP-67 PG Program Listing (Cont)

STEP	KEY ENTRY	KEY CODE	COMMENTS
	−	51	
	RCLE	3415	
	Y^X	3563	
	STO4	3304	
	LSTX	3582	
	R∨	3553	
	−	51	
120	R∧	3554	
	LBL3	312503	
	GSB a	322211	
	RCL1	3401	
	−	51	
	RCL4	3404	
	X ⟷ Y	3552	
	÷	81	
	RCL3	3403	
	X	71	
130	X ⟷ Y	3552	
	X	71	
	STO4	3304	
	LSTX	3582	
	R∨	3553	
	−	51	
	R∧	3554	
	1	01	
	+	61	
	1	01	
140	RCL8	3408	
	−	51	
	R∧	3554	
	X	71	
	EEX	43	
	CHS	42	
	8	08	
	X > Y ?	3281	
	GTO4	2204	
	R∧	3554	
150	R∧	3554	
	GTO3	2203	
	LBL4	312504	
	RCL5	3405	
	STO9	3309	
	RTN	3522	
	LBL a	322511	
	RCL7	3407	
	RCL2	3402	
	X	71	
160	RCL6	3406	
	÷	81	
	STO7	3307	
	STO+8	336108	
	R∧	3554	
	X	71	
	STO+5	336105	
	R∨	3553	
	RCL6	3406	

STEP	KEY ENTRY	KEY CODE	COMMENTS
	1	01	
170	+	61	
	STO6	3306	
	RTN	3522	
180			
190			
200			
210			
220			

LABELS

A N PF→	B SNR dB→	C K→P	D	E Y SUB
a SUB	b X→	c	d	e
0 "1" LOOP	1 Y LOOP	2 BRANCH	3 LOOP	4 BRANCH
5	6	7	8	9

FLAGS

0
1
2
3

SET STATUS

FLAGS	ON	OFF	TRIG		DISP	
0	☐	☐	DEG	☐	FIX	☐
1	☐	☐	GRAD	☐	SCI	☐
2	☐	☐	RAD	☐	ENG	☐
3	☐	☐			n ___	

2. FIXED-THRESHOLD, BARTON ALGORITHM

The program, HP-67 SNR, calculates the required average SNR in dB for a given detection probability and target model. The target model is specified by its probability density function, i.e., nonfluctuating, Rayleigh (i.e., Swerling Case I) or Rayleigh plus an equal power constant component (i.e., Swerling Case II) and by its diversity order, Ne, defined as the number of independent target values within the N samples noncoherently integrated.

Specific user instructions follow:

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS		OUTPUT DATA/UNITS
1	Enter SNR program				
2	(Repeat as desired)	N	↑		
		PD $\geq$ 0.3	↑		
		PF $\leq$ 0.5	D		SNRN dB
	For integration gain if desired:		RCL	0	Gi dB
3	To include collapsing loss or target fluctuation	$1 \leq N_s \leq N$	↑		
		$1 \leq N_e \leq N_s$	f	d	
	Go to step 4, 5 or 6				
4	For nonfluctuating target		A		SNRC dB
5	For Rayleigh target (inc. Cases I and II)		B		SNRF dB
6	For Rayleigh constant target (inc. Cases III & IV)		C		SNRF dB
	(Steps 4, 5 and 6 may be repeated in any desired order)				
	After running each, if desired:		R/S		Ls dB
			R/S		Gd dB
			E		R/R_o

Gd : Diversity gain (dB)
Gi : Integration gain (dB)
Lf : Fluctuation loss (dB)
N : Signal and/or noise samples integrated
Ne : Target diversity within Ns samples
  For Cases I and III : Ne=1
  For Cases II and IV : Ne=Ns
Ns : Samples within N containing signal
PD : Probability of detection
PF : Probability of false alarm
R/Ro : Ratio of detected range to that for which SNRF = 0 dB
SNRC : SNR sample for nonfluctuating target w/collapsing loss (dB)
SNRF : Average SNR per sample for fluctuating target (dB)
SNRN : SNR per sample for nonfluctuating target (dB)

HP-67 SNR Program Listing

STEP	KEY ENTRY	KEY CODE	COMMENTS	STEP	KEY ENTRY	KEY CODE	COMMENTS
001	LBLD	312514			0	00	
	STO3	3303			4	04	
	R∨	3553			4	04	
	STO2	3302		060	8	08	
	R∨	3553			X	71	
	STO1	3301			•	83	
	0	00			9	09	
	STI	3533			9	09	
	RCL2	3402			2	02	
010	2	02			+	61	
	X	71			X	71	
	1	01			1	01	
	X≤Y?	3271			+	61	
	GTO9	2209		070	R∧	3554	
	STI	3533			•	83	
	R∨	3553			2	02	
	LBL9	312509			7	07	
	RCL2	3402			1	01	
	–	51			X	71	
020	fb	322212			2	02	
	DSZ	3133			•	83	
	CHS	42			3	03	
	SPACE	3584			1	01	
	STO4	3304		080	+	61	
	RCL3	3403			X↔Y	3552	
	fb	322212			÷	81	
	RCI4	3404			–	51	
	–	51			RTN	3522	
	↑	41			LBLe	322515	
030	X	71			↑	41	
	2	02			↑	41	
	÷	81			↑	41	
	STO4	3304			9	09	
	RCL1	3401		090	•	83	
	÷	81			2	02	
	fe	322215			+	61	
	STO5	3305			X	71	
	STO8	3308			√	3154	
	RCI4	3404			+	61	
040	fe	322215			2	02	
	STO4	3304			÷	81	
	RCL5	3405			LOG	3153	
	–	51			1	01	
	STO0	3300		100	0	00	
	RCL5	3405			X	71	
	RTN	3522			RTN	3522	
	LBLb	322512			R/S	84	
	↑	41			LBLd	322514	
	X	71			STO7	3307	
050	LN	3152			R∨	3553	
	CHS	42			STO6	3306	
	√	3154			RTN	3522	
	↑	41			LBLA	312511	
	↑	41		110	0	00	
	↑	41			GTO1	2201	
	•	83			LBLB	312512	

REGISTERS

0	1	2	3	4	5	6	7	8	9
Gi dB	N	PD	PF	$-Q^{-1}$, X, SNR	SNRN	Ns	Ne	SNRN dB	
S0	S1	S2	S3	S4	S5	S6	S7	S8	S9

A	B	C	D	E	I

HP-67 SNR Program Listing (Cont)

STEP	KEY ENTRY	KEY CODE	COMMENTS
	RCL3	3403	
	LN	3152	
	RCL2	3402	
	LN	3152	
	÷	81	
	1	01	
	−	51	
120	GTO2	2202	
	LBLC	312513	
	.	83	
	3	03	
	6	06	
	1	01	
	RCL3	3403	
	LOG	3153	
	−	51	
	3	03	
130	.	83	
	2	02	
	7	07	
	RCL2	3402	
	CHS	42	
	1	01	
	+	61	
	√	3154	
	÷	81	
	LSTX	3582	
140	.	83	
	9	09	
	6	06	
	X	71	
	−	51	
	1	01	
	.	83	
	2	02	
	9	09	
	−	51	
150	X	71	
	2	02	
	−	51	
	LBL2	312502	
	LOG	3153	
	1	01	
	0	00	
	X	71	
	RCL4	3404	
	−	51	
160	RCL7	3407	
	÷	81	
	LBL1	312501	
	↑	41	
	↑	41	
	RCL1	3401	
	RCL6	3406	
	÷	81	
	LOG	3153	

STEP	KEY ENTRY	KEY CODE	COMMENTS
	1	01	
170	0	00	
	X	71	
	+	61	
	RCL8	3408	
	+	61	
	STO5	3305	
	R/S	84	
	R↓	3553	
	R/S	84	
	RCL7	3407	
180	1	01	
	−	51	
	X	71	
	R/S	84	
	LBLE	312515	
	RCL5	3405	
	CHS	42	
	4	04	
	0	00	
	÷	81	
190	10^x	3253	
	RTN	3522	
200			
210			
220			

LABELS

A →SNRC dB	B →SNRF dB	C →SNRF dB	D N, PD, PF→	E →R/Ro
a	b Subroutine	c	d Ns↑ Ne	e Subroutine
0	1 Branch	2 Branch	3	4
5	6	7	8	9 Branch

FLAGS

0	
1	
2	
3	

SET STATUS

FLAGS	ON	OFF	TRIG		DISP	
0	☐	☐	DEG	☐	FIX	☐
1	☐	☐	GRAD	☐	SCI	☐
2	☐	☐	RAD	☐	ENG	☐
3	☐	☐			n	____

3. CFAR DETECTION, RECURSIVE SOLUTIONS

These HP-67 programs calculate the detection probability, given the average sample SNR and target model for an adaptive detector threshold which is set proportional to the non-coherent integration of R noise samples. They require an initial calculation of the threshold proportionality constant, T, or equivalently A = 1/(T+1). Each program includes the required T or A iterative calculation. The HP-67 P2C program can also calculate iteratively the required average SNR for a Case II target and a given detection probability. The HP-67 PGC program calculates detection probability for the general chi-squared target model.

Specific user instructions follow:

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS		OUTPUT DATA/UNITS
1	Enter P2C or PGC program	N	↑		
		R	↑		
		PF	A		PF (calc)
	For P2C go to step 2 or 3. For PGC go to step 4.				
2	For P2 do either:	SNR dB	B		P2
	or:	X	f	b	P2
	Repeat for new SNR or X or go to step 3 as desired.				
3	For SNR or X given P2	P2	C		SNR dB
			RCL	A	X
	Repeat or go to step 2 as desired				
4	For PG	SNR dB	B		K
	Enter desired K if different from display	K	R/S		P
	Repeat 4 as desired.				
	After step 2, 3 or 4 do any of the following desired:		RCL	0	PF (calc)
			RCL	1	N
			RCL	2	R
			RCL	3	T
		P2C only	RCL	9	P
			RCL	A	X
			RCL	B	SNR (dB)
	Note: PGC does not store P, X, or SNR		RCL	3	A
	to completion. They may be recalculated	PGC only	RCL	4	Z
	after running as X=Z/N, SNR=10 log X		RCL	E	K

A : 1/(T+1)
K : Chi-squared distribution parameter
N : Signal and/or noise samples integrated
PF : Probability of false alarm
P : Probability of detection for chi-squared target
P0 : Probability of detection for nonfluctuating target
P1-P4 : Probability of detection for Swerling Cases I-IV
R : Noise samples integrated to set threshold
SNR : 10 log X
T : Threshold setting divisor
T0 : Iterative solution start value for T
T20 : Iterative solution start value for T2=T/(1+X)
X : Average sample S/N within N samples

Case 0 : $K \sim 10^5$ (~3 place acc)
Case I : K=1
Case II : K=N
(Genl Rayleigh target : K=Ne*)
Case III : K=2
Case IV : K=2N
(Genl Rayleigh + equal constant target : K=2Ne*)
Weinstock : 0 < K < 1

*Ne: Target diversity within N samples

HP-67 P2C Program Listing

STEP	KEY ENTRY	KEY CODE	COMMENTS
001	LBLA	312511	
	SF2	355102	
	STO4	3304	
	R∨	3553	
	STO2	3302	
	R∨	3553	
	STO1	3301	
	LBL1	312501	
	RCL4	3404	
010	LOG	3153	
	CHS	42	
	RCL1	3401	
	↑	41	
	√	3154	
	−	51	
	LSTX	3582	
	1	01	
	−	51	
	R∧	3554	
020	√	3154	
	+	61	
	LSTX	3582	
	÷	81	
	X↔Y	3552	
	R∧	3554	
	÷	81	
	2	02	
	.	83	
	3	03	
030	÷	81	
	+	61	
	RCL2	3402	
	1	01	
	−	51	
	RCL2	3402	
	.	83	
	9	09	
	2	02	
	2	02	
040	+	61	
	÷	81	
	X	71	
	1	01	
	LSTX	3582	
	−	51	
	RCL1	3401	
	X	71	
	+	61	
	1	01	
050	RCL4	3404	
	RCL2	3402	
	1/X	3562	
	Y^X	3563	
	−	51	
	LSTX	3582	
	÷	81	

STEP	KEY ENTRY	KEY CODE	COMMENTS
	X	71	
	1/X	3562	
	STO3	3303	
060	1	01	
	9	09	
	CHS	42	
	STI	3533	
	LBL0	312500	
	RCL3	3403	
	D	312214	
	RCL5	3405	
	RCL4	3404	
	÷	81	
070	LN	3152	
	STO C	3313	
	RCL6	3406	
	RCL5	3405	
	÷	81	
	RCL2	3402	
	RCL3	3403	
	÷	81	
	−	51	
	÷	81	
080	STO+3	336103	
	RCL C	3413	
	ABS	3564	
	EEX	43	
	CHS	42	
	6	06	
	X ≤ Y ?	3271	
	GTO 0	2200	
	RCL5	3405	
	TF2	357102	
090	STO 0	3300	
	TF3	357103	
	STO 9	3309	
	RTN	3522	
	LBL B	312512	
	1	01	
	0	00	
	÷	81	
	10^x	3253	
	LBL b	322512	
100	STOA	3311	
	LOG	3153	
	1	01	
	0	00	
	X	71	
	STOB	3312	
	RCLA	3411	
	1	01	
	+	61	
	RCL3	3403	
110	X	71	
	D	312214	
	RCL5	3405	

REGISTERS

0	1	2	3	4	5	6	7	8	9
PF	N	R	T	PIN	P_C	Q	T+1	TEMP	P2
S0	S1	S2	S3	S4	S5	S6	S7	S8	S9

A	B	C	D	E	I
X	SNR dB				19

HP-67 P2C Program Listing (Cont)

STEP	KEY ENTRY	KEY CODE	COMMENTS	STEP	KEY ENTRY	KEY CODE	COMMENTS
	STO9	3309					
	RTN	3522		170			
	LBLC	312513					
	SF3	355103					
	STO4	3304					
	RCL3	3403					
	STO8	3308					
120	GSB1	312201					
	RCL3	3403					
	RCL8	3408					
	STO3	3303					
	÷	81		180			
	1	01					
	-	51					
	STOA	3311					
	LOG	3153					
	1	01					
130	0	00					
	X	71					
	STOB	3312					
	RTN	3522					
	LBLD	312514		190			
	↑	41					
	↑	41					
	1	01					
	+	61					
	STO7	3307					
140	÷	81					
	RCL2	3402					
	Y^X	3563					
	STO5	3305					
	0	00		200			
	STO6	3306					
	RCL2	3402					
	X ↔ Y	3552					
	+	61					
	LSTX	3582					
150	R∨	3553					
	X	71					
	RCL7	3407					
	÷	81					
	STO+6	336106		210			
	R∧	3554					
	1	01					
	+	61					
	RCL1	3401					
	X ≤ Y ?	3271					
160	RTN	3522					
	R∨	3553					
	÷	81					
	STO+5	336105					
	LSTX	3582		220			
	GTO(i)	2224					

LABELS					FLAGS
A N, R, PF→	B SNR db→P	C P→SNR dB	D SUB	E	0
a	b X → P	c	d	e	1
0 LOOP	1 LOOP	2	3	4	2 A
5	6	7	8	9	3 C

SET STATUS

FLAGS	ON	OFF	TRIG		DISP	
0	☐	☐	DEG	☐	FIX	☒
1	☐	☐	GRAD	☐	SCI	☐
2	☐	☐	RAD	☐	ENG	☐
3	☐	☐			n	6

HP-67 PGC Program Listing

STEP	KEY ENTRY	KEY CODE	COMMENTS
001	LBLA	312511	
	STO4	3304	
	R↓	3553	
	STO2	3302	
	R↓	3553	
	STO1	3301	
	RCL4	3404	
	LOG	3153	
	CHS	42	
010	RCL1	3401	
	↑	41	
	√	3154	
	-	51	
	LSTX	3582	
	1	01	
	-	51	
	R↑	3554	
	√	3154	
	+	61	
020	LSTX	3582	
	÷	81	
	X ↔ Y	3552	
	R↑	3554	
	÷	81	
	2	02	
	.	83	
	3	03	
	÷	81	
	+	61	
030	RCL2	3402	
	1	01	
	-	51	
	RCL2	3402	
	.	83	
	9	09	
	2	02	
	2	02	
	+	61	
	÷	81	
040	X	71	
	1	01	
	LSTX	3582	
	-	51	
	RCL1	3401	
	X	71	
	+	61	
	1	01	
	RCL4	3404	
	RCL2	3402	
050	1/X	3562	
	Y^X	3563	
	-	51	
	LSTX	3582	
	÷	81	
	X	71	
	1/X	3562	

STEP	KEY ENTRY	KEY CODE	COMMENTS
	STO3	3303	
	LBL0	312500	
	RCL3	3403	
060	↑	41	
	↑	41	
	1	01	
	+	61	
	STO7	3307	
	÷	81	
	RCL2	3402	
	Y^X	3563	
	STO0	3300	
	0	00	
070	STO6	3306	
	LBL3	312503	
	RCL2	3402	
	X ↔ Y	3552	
	+	61	
	LSTX	3582	
	R↓	3553	
	X	71	
	RCL7	3407	
	÷	81	
080	STO+6	336106	
	R↑	3554	
	1	01	
	+	61	
	RCL1	3401	
	X ≤ Y ?	3271	
	GTO7	2207	
	R↓	3553	
	÷	81	
	STO+0	336100	
090	LSTX	3582	
	GTO3	2203	
	LBL7	312507	
	RCL0	3400	
	RCL4	3404	
	÷	81	
	LN	3152	
	STOC	3313	
	RCL6	3406	
	RCL0	3400	
100	÷	81	
	RCL2	3402	
	RCL3	3403	
	÷	81	
	-	51	
	÷	81	
	STO+3	336103	
	RCLC	3413	
	ABS	3564	
	EEX	43	
110	CHS	42	
	6	06	
	X ≤ Y ?	3271	

REGISTERS

0	1	2	3	4	5	6	7	8	9
PFC(PMS)	N	R	T, A	PF, Z	XB	QMS, M	T+1	PMS	1-XBS
S0	S1	S2	S3	S4	S5	S6	S7	S8	S9

A	B	C	D	E	I
		LN(PFC/PF)	V	K	

HP-67 PGC Program Listing (Cont)

STEP	KEY ENTRY	KEY CODE	COMMENTS
	GTO0	2200	
	RCL3	3403	
	1	01	
	+	61	
	1/X	3562	
	STO3	3303	
	RCL0	3400	
120	RTN	3522	
	LBLB	312512	
	1	01	
	0	00	
	÷	81	
	10^x	3253	
	RCL1	3401	
	X	71	
	STO4	3304	
	RCLE	3415	
130	R/S	84	
	STOE	3315	
	1	01	
	RCL3	3403	
	-	51	
	RCL2	3402	
	Y^x	3563	
	STO7	3307	
	STO8	3308	
	1	01	
140	STO6	3306	
	LBL1	312501	
	RCL1	3401	
	X ≤ Y	3271	
	GTO2	2202	
	a	322211	
	GTO1	2201	
	LBL2	312502	
	1	01	
	RCL4	3404	
150	RCL4	3404	
	RCLE	3415	
	+	61	
	÷	81	
	STOD	3314	
	-	51	
	RCLE	3415	
	Y^x	3563	
	STO5	3305	
	LSTX	3582	
160	1	01	
	-	51	
	RCL8	3408	
	R∨	3553	
	R∨	3553	
	-	51	
	STO9	3309	
	R∧	3554	
	LBL6	312506	

STEP	KEY ENTRY	KEY CODE	COMMENTS
	R∨	3553	
170	R∨	3553	
	a	322211	
	R∨	3553	
	RCL9	3409	
	RCL7	3407	
	X	71	
	+	61	
	RCL6	3406	
	RCL1	3401	
	-	51	
180	R∧	3554	
	X ↔ Y	3552	
	+	61	
	LSTX	3582	
	÷	81	
	RCLD	3414	
	X	71	
	RCL5	3405	
	X	71	
	STO5	3305	
190	STO-9	335109	
	CLX	44	
	1	01	
	RCL8	3408	
	-	51	
	EEX	43	
	8	08	
	X	71	
	1/X	3562	
	RCL9	3409	
200	X > Y ?	3281	
	GTO6	2206	
	R∨	3553	
	R∨	3553	
	RTN	3522	
	LBLa	322511	
	RCL2	3402	
	RCL6	3406	
	+	61	
	1	01	
210	-	51	
	RCL6	3406	
	÷	81	
	RCL3	3403	
	X	71	
	RCL7	3407	
	X	71	
	STO7	3307	
	STO+8	336108	
	R∨	3553	
220	RCL6	3406	
	1	01	
	+	61	
	STO6	3306	
	RTN	3522	

LABELS

A N↑R→	B SNR dB+K→	C	D SUB	E
a SUB	b	c	d	e
0 LOOP	1	2	3 LOOP	4 LOOP
5 BRANCH	6 LOOP	7 BRANCH	8	9

FLAGS

0	
1	
2	
3	

SET STATUS

FLAGS ON OFF	TRIG	DISP
0 ☐ ☐	DEG ☐	FIX ☐
1 ☐ ☐	GRAD ☐	SCI ☐
2 ☐ ☐	RAD ☐	ENG ☐
3 ☐ ☐		n___

SECTION VII

ILLUSTRATIVE EXAMPLES

1. HP-65 FIXED-THRESHOLD, RECURSIVE

Program	Step	Input	Key	Output	Time (m:s)
Y-P2	1	N=10	↑		
		$PF=10^{-6}$	A	Y=32.710341	0:28
	2	$X=\sqrt{10}$	B	P2=0.733987	0:08
	3	P2=0.9	A	Y2=6.221305	0:45
			R/S	X=4.257794	0:01
P0	4	$X=\sqrt{10}$	B	P0=0.853317	2:23
P1	4	$X=\sqrt{10}$	B	P1=0.485543	0:18
P3	4	$X=\sqrt{10}$	B	P3=0.569375	0:20
P4	4	$X=\sqrt{10}$	B	P4=0.781789	0:33
P6	7	$K=10^5$	↑		
		$X=\sqrt{10}$	B	P0≈0.853297	2:19
	7	K=1	↑		
		$X=\sqrt{10}$	B	P1=0.485543	2:35
	7	K=10	↑		
		$X=\sqrt{10}$	B	P2=0.733987	2:31
	7	K=2	↑		
		$X=\sqrt{10}$	B	P3=0.569375	2:34
	7	K=20	↑		
		$X=\sqrt{10}$	B	P4=0.781789	2:27

2. HP-65 FIXED-THRESHOLD, BARTON

Program	Step	Input	Key	Output	Time (m:s)
SNRN	2	N=10	STO 1		
		PD=0.75	STO 2		
		$PF=10^{-6}$	STO 3		
			A	SNRN=4.34	0:09
			-	G_i=7.89	
	3	N_s=10	STO 6		
		N_e=10	STO 7		
	4		A	SNRC=4.34	0:01
	5		B	SNRF=4.79	0:03
	6		C	SNRF=4.57	0:04
			R/S	L_f=0.22	0:01
			R/S	G_d=2.05	0:01
			E	R/R_o=0.769	0:01

3. HP-65 CFAR DETECTION

Program	Step	Input	Key	Output	Time (m:s)
T0	1	N=10	↑		
		R=16	↑		
		$PF=10^{-6}$	A	T0=0.224789	0:04
P2C	2		R/S	T=0.234428	0:41
	3	SNR=10	B	P2=0.867652	0:13
T0	4	P2=0.9	C	T20=2.207242	0:04
P2C	5		R/S	SNR=10.539693	1:10
PGC(1)	6		R/S	A=0.810092	0:41*
	7	SNR=10	↑		
		K=10	A	1	0:01
PGC(2)	8		R/S	P2=0.867652	9:57
PGC(1)	7	SNR=10	↑		
		K=1	A	1	0:01
PGC(2)	8		R/S	P1=0.564214	10:42

*0:14 if step 2 has been run

4. HP-67 FIXED-THRESHOLD, RECURSIVE

Program	Step	Input	Key	Output	Time (m:s)
Any below	1	N=10	↑		
		PF=10^{-6}	A	PF=0.000001	0:32
P1-P2	2	SNR=5	B	P2=0.733987	0:11
	3	P2=0.9	C	SNR=6.291847	0:56
	4	SNR=5	D	P1=0.485543	0:24
P0	2	SNR=5	B	P0:0.853317	2:46
P3	2	SNR=5	B	P3=0.569375	0:08
P4	2	SNR=5	B	P4=0.781789	0:39
PG	5	SNR=5	B		
	6	K=10^5	C	P0≈0.853298	2:40
	6	K=1	C	P1=0.485543	2:59
	6	K=10	C	P2=0.733987	2:55
	6	K=2	C	P3=0.569375	2:58
	6	K=20	C	P4=0.781789	2:51

5. HP-67 FIXED-THRESHOLD, BARTON

Program	Step	Input	Key	Output	Time (m:s)
SNR	2	N=10	↑		
		PD=0.75	↑		
		PF=10^{-6}	D	SNRN=4.34	0:13
	3	N_s=10	↑		
		N_e=10	fd		
	4		A	SNRC=4.34	0:02
	5		B	SNRF=4.79	0:05
	6		C	SNRF=4.57	0:06
			R/S	L_f=0.22	0:01
			R/S	G_d=2.05	0:01
			E	R/R_o=0.769	0:01

6. HP-67 CFAR DETECTION

Program	Step	Input	Key	Output	Time (m:s)
P2C or PGC	1	N=10			
		R=16			
		PF=10^{-6}	A	PF=0.000001	1:01
P2C	2	SNR=10	B	P2=0.867652	0:14
	3	P2=0.9	C	SNR=10.339693	1:15
PGC	4	SNR=10	B		
		K=10	R/S	P2=0.867552	10:24
	4	SNR=10	B		
		K=1	R/S	P1=0.564214	11:11

SECTION VIII

REFERENCES

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2. Swerling, P., "Probability of Detection for Fluctuating Targets", RAND Corp. Res. Memo RM-1217, March 17, 1954 (reprinted in IEEE Transactions IT-6, No. 2, April 1960, pp 269 - 308).

3. Barton, D.K., "Simple Procedures for Radar Detection Calculations", IEEE Transactions AES-5, No. 5, September 1969, pp 837 - 46.

4. Shnidman, D.A., "Evaluation of Probability of Detection for Several Target Fluctuation Models", Lincoln Lab Tech Note 1975 - 35, July 9, 1975, ADA 013733.

5. Mitchell, R.L. and Walker, J.F., "Recursive Methods for Computing Detection Probabilities", IEEE Transactions AES-7, No. 4, July 1971, pp 671 - 676.

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7. Cann, A.J., "Simple Radar Detection Calculation," IEEE Transactions AES-8, No. 1, January 1972, pp 73 - 74.

8. Mayer, H.A. and Meyer, D.P., "Chi-Square Target Models of Low Degrees of Freedom", IEEE Transactions AES-11, No. 5, September 1975, p 694 - 707.

9. Abramowitz, M. and Stegun, I.A., Handbook of Mathematical Functions, Nat. Bureau of Stds, Applied Math Series 55, June 1964, p 933, 26.2.22.

10. DiFranco, J.V. and Rubin, W.L., Radar Detection, Prentice Hall, Englewood Cliffs, N.J., 1968, p 315, Eqn 9.5-8b, with R=2D.

SUPPLEMENTARY

INFORMATION

AD A071609

ERRATA SHEET

September 12, 1979

Please make the following changes to TIS R79EMH5:

p. 2-5	Change reference number for Shnidman from 2 to 4.
p. 3-1	Equation (3-1) center should read $X = \frac{1}{2N}\left[Q^{-1}(PF) - Q^{-1}(PD)\right]^2$
p. 3-1	Change reference number for Cann from 8 to 7.
p. 5-1	Step 5 output: Change "D" to "P".
p. 5-1	In instruction table - step 7, first key block after K should contain "↑".
p. 5-2	Step 25, Comment should read: $2.3\sqrt{L}\,(\sqrt{L} + \sqrt{N} - 1)$, $N - \sqrt{N}$
p. 5-3	Step 3, change "3302" to "3303".
p. 5-4	Step 4, change "3403" to "3402".
p. 5-4	Step 19, change "3302" to "3402".
p. 5-7	On program line 52, change comment from "XB · V/B" to "XB · V/(M-N)".
p. 5-7/ 5-8	Step 5, change "3303" to "3304".
p. 5-9	Instr. step 5 should read: "For Rayleigh tgt. (Inc Cases I & II)
p. 5-10	Line 21 comment should read $\left[Q^{-1}(PF) - Q^{-1}(PD)\right]$; Comment column line 17, remove minus sign.
p. 5-13	In nomenclature table: Case 0: K 10^5
p. 6-2	Step 5, "P6" should be "PG".
p. 6-2	Add to step 1, 1st block , insert ↑ .
p. 6-5	On program line 089, change "X" to "+".
p. 7-1	Change last item under Program from P6 to PG.
p. 7-1	Line 1, step 1, change " ↑ " to "STO1".
p. 7-2	Par. 2, step 3, add <u>SNRF</u> in <u>Program</u> Column.
p. 7-4	Step 1 in Key column, add ↑ in Key Column after N = 10 and R = 16.