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Airplane Stability Calculations With a Card Programmable Pocket Calculator

Windsor L. Sherman

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ERRATA

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AIRPLANE STABILITY CALCULATIONS WITH A CARD PROGRAMMABLE POCKET CALCULATOR

Windsor L. Sherman

August 1978

Please make the following corrections:

Page 15: Sentence after equation (11) should read as follows:

Equations (9) and (10) were programmed for the calculator and the program is given in appendix B.

Page 16: Equation (16) should read as follows:

$$Re(y) = S + T - \frac{b_2}{3}$$

Page 24, Last sentence: Change step 49 to step 45.

Page 25: Step 100 should read as follows:

STO×9 $(g\sigma_T/2U_{SS}) \sin 2\gamma_{SS}$

Page 26, Step 105: Change - to +
Step 141: Change RCL8 to RCLB

Page 29: Delete the last sentence.

Page 49: In column headed "Output," change the values of a_3, a_2, a_1, a_0, and a_{12} to

a_3 = 1.3980958

a_2 = 1.1093007

a_1 = -0.0098076

a_0 = -0.0211448

a_{12} = 0.0373094

ISSUED NOVEMBER 1978

ERRATA

NASA Technical Memorandum 78737

DEVELOPMENT OF A NONLINEAR SWITCHING FUNCTION AND ITS APPLICATION TO STATIC LIFT CHARACTERISTICS OF STRAIGHT WINGS

Donald E. Hewes

September 1978

Page 5: Equation (3) should read

$$x_{10} = x_e\left(\frac{\ln e}{\ln 10}\right)^{1/2} = x_e\left(\frac{1}{\ln 10}\right)^{1/2}$$

ISSUED NOVEMBER 1978

NASA Technical Memorandum 78678

Airplane Stability Calculations With a Card Programmable Pocket Calculator

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Hampton, Virginia

National Aeronautics
and Space Administration

**Scientific and Technical
Information Office**

1978

SUMMARY

Programs are presented for calculating airplane stability characteristics with a card programmable pocket calculator. These calculations include eigenvalues of the characteristic equations of lateral and longitudinal motion as well as stability parameters such as the time to damp to one-half amplitude or the damping ratio. The effects of wind shear are included. Background information and the equations programmed are given. The programs are written for the International System of Units, the dimensional form for the stability derivatives, and stability axes. In addition to the programs for stability calculations, an unusual and short program is included for the Euler transformation of coordinates used in airplane motions. The programs have been written for a Hewlett Packard HP-67 calculator. However, the use of this calculator does not constitute an endorsement of the product by the National Aeronautics and Space Administration.

INTRODUCTION

Over the past several years, the programmable pocket calculator has developed into a highly sophisticated device that has almost computer characteristics. Because of its sophistication, the newer models are capable of being programmed to make very complicated calculations. Since different logics are used in programmable calculators and since the available keyboard instructions vary with models of different manufacturers, it is necessary to identify the make and model of the calculator for which a program is written. The airplane stability programs presented in this paper were written for a Hewlett Packard HP-67 card programmable calculator; however, its use and identification in this report does not constitute an endorsement of the product by the National Aeronautics and Space Administration.

Programs are given for the calculation of the coefficients of the airplane lateral and longitudinal characteristic equations, the eigenvalues, and the stability parameters such as the time to damp to one-half amplitude or the damping ratio. In addition, a unique coordinate transformation program is given for transformations between inertial axes and airplane body axes. This program requires very few program steps and may be useful as part of a larger program. The equations on which the programs are based are given so that the programs can be readily adapted to other calculators that have sufficient program capacity.

The programs presented herein evolved during the study of wind shear and its effect on airplane stability and control. These programs proved useful in making stability calculations in this study and should be of use in other investigations.

SYMBOLS

A	aspect ratio
$a_0, a_1, \ldots, a_5$	coefficients of characteristic equations
$a_{12}, a_{13}, a_{14}, \ldots$	elements of longitudinal stability determinant
b	wing span
b_2, b_1, b_0	coefficients of resolvent cubic
C_D	drag coefficient, $\dfrac{D}{\rho S U_{ss}^2/2}$
$C_{D,o}$	drag coefficient for $C_L = 0$
$C_{D\alpha}$	$= \dfrac{\partial C_D}{\partial \alpha}$
C_L	lift coefficient, $\dfrac{L}{\rho S U_{ss}^2/2}$
$C_{L,o}$	lift coefficient at zero angle of attack
$C_{L\alpha}$	$= \dfrac{\partial C_L}{\partial \alpha}$
$C_{L\dot{\alpha}}$	$= \dfrac{\partial C_L}{\partial \dot{\alpha}}$
$C_{L\dot{\theta}}$	$= \dfrac{\partial C_L}{\partial \dot{\theta}}$
C_l	rolling-moment coefficient, $\dfrac{M_X}{\rho S b U_{ss}^2/2}$
C_{l_p}	$= \dfrac{\partial C_l}{\partial p}$
C_{l_r}	$= \dfrac{\partial C_l}{\partial r}$

C_{l_β} $= \dfrac{\partial C_l}{\partial \beta}$

$C_{l_{\dot\beta}}$ $= \dfrac{\partial C_l}{\partial \dot\beta}$

C_{l_ϕ} $= \dfrac{\partial C_l}{\partial \phi}$

C_m pitching-moment coefficient, $\dfrac{M_Y}{\rho S\bar{c}U_{SS}^2/2}$

$C_{m,o}$ total pitching-moment coefficient at zero angle of attack

C_{m_α} $= \dfrac{\partial C_m}{\partial \alpha}$

$C_{m_{\dot\alpha}}$ $= \dfrac{\partial C_m}{\partial \dot\alpha}$

$C_{m_{\dot\theta}}$ $= \dfrac{\partial C_m}{\partial \dot\theta}$

C_n yawing-moment coefficient, $\dfrac{M_Z}{\rho SbU_{SS}^2/2}$

C_{n_p} $= \dfrac{\partial C_n}{\partial p}$

C_{n_r} $= \dfrac{\partial C_n}{\partial r}$

C_{n_β} $= \dfrac{\partial C_n}{\partial \beta}$

$C_{n_{\dot\beta}}$ $= \dfrac{\partial C_n}{\partial \dot\beta}$

C_{n_ϕ}	$= \dfrac{\partial C_n}{\partial \phi}$	
C_T	thrust coefficient	
C_{T_u}	$= \dfrac{\partial C_T}{\partial u}$	
C_Y	side-force coefficient, $\dfrac{F_Y}{\rho S U_{ss}^2/2}$	
C_{Y_p}	$= \dfrac{\partial C_Y}{\partial p}$	
C_{Y_r}	$= \dfrac{\partial C_Y}{\partial r}$	
C_{Y_β}	$= \dfrac{\partial C_Y}{\partial \beta}$	
$C_{Y_{\dot\beta}}$	$= \dfrac{\partial C_Y}{\partial \dot\beta}$	
c_{11}, c_{21}, c_{30} $b_{11}, b_{12}, b_{13}, \ldots$	terms in lateral stability determinant	
$\bar{c}$	mean aerodynamic chord	
D	drag	
F_T	thrust	
$F_{T,tr}$	trim thrust	
F_{T_u}	$= \dfrac{\partial F_T}{\partial u}$	
F_X, F_Y, F_Z	forces along X, Y, and Z stability axis	
$F_{X_{\delta_e}}$	$= \dfrac{\partial F_X}{\partial \delta_e}$	

$F_{Y\delta_a}$ $= \dfrac{\partial F_Y}{\partial \delta_a}$

$F_{Y\delta_r}$ $= \dfrac{\partial F_Y}{\partial \delta_r}$

$F_{Z\delta_e}$ $= \dfrac{\partial F_Z}{\partial \delta_e}$

g acceleration of gravity

I_X, I_Y, I_Z moments of inertia, stability axes

I_{XZ} product of inertia, stability axes

Im( ) imaginary part of complex root

k_X, k_Y, k_Z, k_{XZ} radii of gyration, stability axes

L lift

M_X, M_Y, M_Z moments about X, Y, and Z stability axes

$M_{X\delta_a}$ $= \dfrac{\partial M_X}{\partial \delta_a}$

$M_{X\delta_r}$ $= \dfrac{\partial M_X}{\partial \delta_r}$

$M_{Y\delta_e}$ $= \dfrac{\partial M_Y}{\partial \delta_e}$

$M_{Z\delta_a}$ $= \dfrac{\partial M_Z}{\partial \delta_a}$

$M_{Z\delta_r}$ $= \dfrac{\partial M_Z}{\partial \delta_r}$

m mass

N_D number of cycles to double amplitude

$N_{1/2}$ number of cycles to damp to one-half amplitude

p	rolling velocity
R_*	radius
Re( )	real part of complex root
Re(y)	real root of resolvent cubic
r	yawing velocity
S	wing area
t	period
t_D	time to double amplitude
$t_{1/2}$	time to damp to one-half amplitude
U_{ss}	steady-state velocity
U_w	wind velocity
u_{pr}	perturbation velocity
u_w'	wind shear gradient
w_w'	updraft-downdraft gradient
X,Y,Z	stability axes
X_b,Y_b,Z_b	airplane body axes
X_e,Y_e,Z_e	Earth-fixed axes
X_{sp},Y_{sp},Z_{sp}	space axes
x,y,z	general variables
x_b,y_b,z_b	body axis coordinates
x_{sp},y_{sp},z_{sp}	space axis coordinates
α_{pr}	perturbation angle of attack
α_{tr}	trim angle of attack
$\left.\begin{matrix}\alpha_1,\alpha_2\\ \alpha_3,\alpha_4\end{matrix}\right\}$	real roots
β	sideslip angle
γ_{pr}	perturbation flight-path angle

γ_{ss}	steady-state flight-path angle
Δ	logarithmic decrement
δ_a	aileron deflection
δ_e	elevator deflection
δ_r	rudder deflection
$\varepsilon_1,\varepsilon_2$	angles
ζ	damping ratio
θ_{tr}	trim pitch angle
ρ	atmospheric density
σ_T	$= \sigma_u + \sigma_w$
σ_u	$= \dfrac{U_{ss}u_w'}{g}$
σ_w	$= \dfrac{U_{ss}w_w'}{g}$
ψ,θ,ϕ	airplane yaw (heading), pitch, and roll angles, respectively
ω_n	undamped circular frequency

Dot over a symbol indicates differentiation with respect to time.

EQUATIONS PROGRAMMED AND PROGRAM DESCRIPTIONS

Six programs are presented in this paper. The first three calculate the elements of the lateral and longitudinal stability determinants and the coefficients of the characteristic equations. In addition, program 3 extracts a real root of a fifth-order polynomial when required. Programs 4 and 5 complete the root extraction process and calculate the stability parameters. Program 6 implements the Euler angle transformation by using the polar-rectangular keys found on calculators.

Programs 1, 2, and 3 are written for the International System of Units, stability axes (fig. 1), and the dimensional form of the stability derivatives. The equations programmed are the linearized form of the equations of motion derived in appendix A of reference 1; thus, the effects of wind shear are included.

In deriving these equations, head winds and updrafts were taken as negative. Thus, a positive u_w' will change a head wind into a tail wind, and a positive w_w' will change an updraft into a downdraft. The signs of u_w' and w_w' set the signs of σ_u and σ_w; u_w' is a gradient with altitude and w_w' is a gradient along the flight path.

In writing the programs, the following conventions were used for the labels:

(1) Capital letters (A to E) are program labels

(2) Lower-case letters (a to e) are subroutine labels

(3) Numbers (0 to 9) are used for all other labels

Table I summarizes the programs presented in this paper. The key entries given in appendixes A to F are the standard HP-67 key entries given in the owner's manual. Check cases for all programs are given in appendix G.

TABLE I.- SUMMARY OF PROGRAMS

Program	Description	Key entries given in
1	Calculates the elements of longitudinal stability determinant and normalized coefficients for characteristic equation	Appendix A
2	Calculates the elements of lateral stability determinant and starts calculating coefficients of the characteristic equation	Appendix B
3	Label A completes calculating coefficients of characteristic equations of lateral motion; label B calculates a real root of a fifth-order polynomial and reduces the fifth-order polynomial to a fourth-order one; $t_{1/2}$ or t_D for the real root determined; label B can be used as a stand-alone program	Appendix C
4	Uses Ferrari's method to calculate the roots of a fourth-order polynomial and can be used as a stand-alone program; will also determine roots of cubic, quadratic, and first-order equations	Appendix D
5	Calculates stability parameters such as $t_{1/2}$, t_D, and $N_{1/2}$	Appendix E
6	Uses the polar-rectangular transformations of the calculator to implement the Euler transformation between space and body axes or body and space axes; this method saves about 57 program steps when compared with the more usual methods of programming	Appendix F

Programs 1 and 2 give solutions from an equilibrium flight condition. There are six parameters, U_{ss}, γ_{ss}, α_{tr}, $F_{T,tr}$, σ_T, and σ_w, that must be adjusted correctly to obtain the equilibrium flight condition. There are two equations to accomplish this adjustment. Programs 1 and 2 were set up in the following manner. The parameters U_{ss}, γ_{ss}, σ_T, and σ_w are specified by the user. The program calculates α_{tr}, assuming that $F_{T,tr}$ is 0. For the flight condition U_{ss} = 77.12 m/sec, γ_{ss} = -0.05236 rad, σ_T = 2.0, and σ_w = 0.0, the error introduced in α_{tr} by this method is 0.00081 rad, which is considered acceptable. If it is desired to monitor the calculated value of α_{tr}, insert a pause after step 45 of program 1.

Program 1

The linearized equation of longitudinal motion is in symbolic form

$$\begin{bmatrix} \frac{d}{dt} + a_{12} & a_{21} & a_{31} \\ a_{13} & a_{22}\frac{d}{dt} + a_{23} & a_{32}\frac{d}{dt} + a_{33} \\ a_{14} & \left(\frac{d}{dt}\right)^2 + a_{25}\frac{d}{dt} + a_{26} & \left(\frac{d}{dt}\right)^2 + a_{35}\frac{d}{dt} \end{bmatrix} \begin{bmatrix} u \\ \alpha_{pr} \\ \gamma_{pr} \end{bmatrix} = \begin{bmatrix} F_{X_{\delta_e}} \\ F_{Z_{\delta_e}} \\ M_{Y_{\delta_e}} \end{bmatrix} \delta_e \tag{1}$$

The characteristic equation for longitudinal stability is obtained from the determinant of the 3 × 3 matrix and has the form

$$a_4\left(\frac{d}{dt}\right)^4 + a_3\left(\frac{d}{dt}\right)^3 + a_2\left(\frac{d}{dt}\right)^2 + a_1\frac{d}{dt} + a_0 = 0 \tag{2}$$

where a_0 to a_4 are given by

$$a_4 = a_{22} - a_{32} \tag{3a}$$

$$a_3 = a_{12}(a_{22} - a_{32}) + (a_{23} - a_{33} - a_{25}a_{32} + a_{22}a_{35}) \tag{3b}$$

$$a_2 = (a_{23}a_{35} - a_{25}a_{33} - a_{26}a_{32}) + a_{12}(a_{23} - a_{33} - a_{25}a_{32} + a_{22}a_{35}) + a_{13}(a_{31} - a_{21}) \tag{3c}$$

$$a_1 = a_{12}(a_{23}a_{35} - a_{25}a_{33} - a_{26}a_{32}) + a_{31}(a_{13}a_{25} - a_{14}a_{22}) - a_{21}(a_{13}a_{35} - a_{14}a_{32}) - a_{26}a_{33} \tag{3d}$$

$$a_0 = a_{33}(a_{21}a_{14} - a_{26}a_{12}) + a_{31}(a_{13}a_{26} - a_{14}a_{23}) \tag{3e}$$

and a_{12}, a_{13}, etc., are given by

$$\left.\begin{aligned}
a_{12} &= -\frac{g\sigma_T}{2U_{ss}} \sin 2\gamma_{ss} + \left(C_{D,o} + \frac{C_L{}^2}{\pi A}\right)k_1 - \frac{F_{T_u}}{m} \\
a_{13} &= C_L k_1 - \frac{g}{U_{ss}}(\sigma_T \sin^2 \gamma_{ss} - \sigma_w) \\
a_{14} &= -\left(C_{m,o} + C_{m_\alpha}\alpha_{tr}\right)k_2 \\
a_{21} &= C_{D_\alpha}k_3 \\
a_{22} &= \left(C_{L_{\dot{\alpha}}} + C_{L_{\dot{\theta}}}\right)k_3 \\
a_{23} &= C_{L_\alpha}k_3 \\
a_{25} &= -\left(C_{m_{\dot{\theta}}} + C_{m_{\dot{\alpha}}}\right)k_4 \\
a_{26} &= -C_{m_\alpha}k_4 \\
a_{31} &= g(\cos \gamma_{ss} - \sigma_T \cos 2\gamma_{ss}) \\
a_{32} &= -U_{ss} + C_{L_{\dot{\theta}}}k_3 \\
a_{33} &= g(\sin \gamma_{ss} - \sigma_T \sin 2\gamma_{ss}) \\
a_{35} &= -C_{m_{\dot{\theta}}}k_4
\end{aligned}\right\} \tag{4}$$

where $k_1 = \frac{\rho S U_{ss}}{m}$, $k_2 = \frac{\rho S \bar{c} U_{ss}}{I_Y}$, $k_3 = \frac{\rho S U_{ss}^2}{2m}$, and $k_4 = \frac{\rho S \bar{c} U_{ss}^2}{2 I_Y}$. In addition to the foregoing equations, the following equations are needed to calculate the values of C_L, C_D, and α_{tr} at trim:

$$C_L = \frac{2mg}{\rho S U_{ss}^2}(\sigma_T \sin^2 \gamma_{ss} - \sigma_w + \cos \gamma_{ss}) \tag{5a}$$

$$C_D = C_{D,o} + \frac{{C_L}^2}{\pi A} \tag{5b}$$

$$\alpha_{tr} = \frac{C_L - C_{L,o}}{C_{L_\alpha}} \tag{5c}$$

Because large changes in forward speed are encountered in wind shear, the effects of the u stability derivatives not normally accounted for are included in this program. This was done in the following manner:

$$D_u = \frac{\partial D}{\partial u} = \left(C_{D,o} + \frac{{C_L}^2}{\pi A}\right)k_1 \qquad \text{(used in eq. 4)}$$

$$L_u = \frac{\partial L}{\partial u} = C_L k_1 \qquad \text{(used in eq. 4)}$$

$$M_{Y_u} = \frac{\partial M_Y}{\partial u} = \left(C_{m,o} + C_{m_\alpha}\alpha_{tr}\right)k_2 \qquad \text{(used in eq. 4)}$$

Equations (3), (4), and (5) were programmed to calculate the coefficients of the characteristic equation, which is equation (2). The key codes for program 1 are given in appendix A.

The program destroys the original input data but preserves the coefficients of the determinant in the secondary registers. The principal output is the normalized coefficients of the characteristic equation which are stored in R_0, R_1, R_2, and R_3.

Program 2

The linearized equations of lateral motion with the effects of wind shear included are, in symbolic form,

$$\begin{bmatrix} C_{11}\dfrac{d}{dt} + b_{13} & C_{21}\dfrac{d}{dt} + b_{22} & C_{30}\dfrac{d}{dt} + b_{31} \\ \left(\dfrac{d}{dt}\right)^2 + b_{14}\dfrac{d}{dt} + b_{15} & b_{42}\left(\dfrac{d}{dt}\right)^2 + b_{23}\dfrac{d}{dt} & b_{32}\dfrac{d}{dt} + b_{33} \\ b_{43}\left(\dfrac{d}{dt}\right)^2 + b_{16}\dfrac{d}{dt} + b_{17} & \left(\dfrac{d}{dt}\right)^2 + b_{24}\dfrac{d}{dt} & b_{34}\dfrac{d}{dt} + b_{35} \end{bmatrix} \begin{bmatrix} \phi \\ \psi \\ \beta \end{bmatrix} = \begin{bmatrix} F_{Y_{\delta_a}} & F_{Y_{\delta_r}} \\ M_{X_{\delta_a}} & M_{X_{\delta_r}} \\ M_{Z_{\delta_a}} & M_{Z_{\delta_r}} \end{bmatrix} \begin{bmatrix} \delta_a \\ \delta_r \end{bmatrix} \tag{6}$$

and are based on equations (A5), (A8), and (A9) of reference 1. In linearizing these equations, it was assumed that no wind gradient existed in the Y_e derivative in Earth axes. If the wind gradients are zero (i.e., no wind shear), these equations reduce to the standard form of the linearized equations of lateral motion that are given in many standard works, such as reference 2. The equations are valid in the interval $-0.17453 \leqq \gamma_{ss} \leqq 0.17453$.

The characteristic equation is obtained from the 3×3 matrix on the left-hand side of equation (6) and has the form

$$a_5\left(\frac{d}{dt}\right)^5 + a_4\left(\frac{d}{dt}\right)^4 + a_3\left(\frac{d}{dt}\right)^3 + a_2\left(\frac{d}{dt}\right)^2 + a_1\frac{d}{dt} + a_0 = 0 \tag{7}$$

for $\sigma_T \neq 0$.

When $\sigma_T = 0$, the a_0 term in equation (7) becomes 0. Equation (7) now has one zero root and four finite roots and is solved as a quartic. Program 2 tests equation (7) and informs the user if a fourth- or fifth-degree polynomial is present. The coefficients a_0 to a_5 are given by

$$a_5 = C_{30}(1 - b_{43}b_{42}) \tag{9a}$$

$$a_4 = C_{11}(b_{42}b_{34} - b_{32}) - C_{21}(b_{34} - b_{43}b_{32}) + b_{31}(1 - b_{43}b_{42}) + C_{30}(b_{24} + b_{14} - b_{43}b_{23} - b_{16}b_{42}) \tag{9b}$$

$$a_3 = b_{13}(b_{42}b_{34} - b_{32}) - b_{22}(b_{34} - b_{43}b_{32}) + C_{11}(b_{42}b_{35} + b_{23}b_{34} - b_{24}b_{32} - b_{33}) - C_{21}(b_{35} + b_{14}b_{34} - b_{43}b_{33} - b_{16}b_{32}) + b_{31}(b_{24} + b_{14} - b_{43}b_{23} - b_{16}b_{42}) + C_{30}(b_{15} + b_{14}b_{24} - b_{16}b_{23} - b_{17}b_{42}) \qquad (9c)$$

$$a_2 = C_{11}(b_{23}b_{35} - b_{24}b_{33}) + b_{13}(b_{42}b_{35} + b_{23}b_{34} - b_{24}b_{32} - b_{33}) - C_{21}(b_{14}b_{35} + b_{15}b_{34} - b_{16}b_{33} - b_{17}b_{32}) + C_{30}(b_{15}b_{24} - b_{17}b_{23}) - b_{22}(b_{35} + b_{14}b_{34} - b_{43}b_{33} - b_{16}b_{32}) + b_{31}(b_{15} + b_{14}b_{24} - b_{16}b_{23} - b_{17}b_{42}) \qquad (9d)$$

$$a_1 = b_{13}(b_{23}b_{35} - b_{24}b_{33}) - b_{22}(b_{14}b_{35} + b_{15}b_{34} - b_{16}b_{33} - b_{17}b_{32}) - C_{21}(b_{15}b_{35} - b_{17}b_{33}) + b_{31}(b_{15}b_{24} - b_{17}b_{23}) \qquad (9e)$$

$$a_0 = -b_{22}(b_{15}b_{35} - b_{17}b_{33}) \qquad (9f)$$

The C and b terms that are used to generate the coefficients of the characteristic equation are given by

$$b_{11} = 0.0 \qquad (10a)$$

$$b_{12} = -C_{Y_p}k_5 \qquad (10b)$$

$$b_{13} = -\frac{g\sigma_w}{U_{ss}^2} - \frac{g}{U_{ss}}\cos\gamma_{ss} \qquad (10c)$$

$$b_{14} = -C_{l_p}k_6 \qquad (10d)$$

$$b_{15} = u_w'\left(-C_{l_r}k_6\right) = C_{l_\phi}k_6 \qquad (10e)$$

$$b_{16} = -C_{n_p}k_7 \qquad (10f)$$

$$b_{17} = u_w'\left(-C_{n_r}k_7\right) = C_{n_\phi}k_7 \qquad (10g)$$

$$b_{21} = -C_{Y_r}k_5 \qquad (10h)$$

$$b_{22} = \frac{g(\sigma_u + \sigma_w)}{2U_{ss}^2} \sin 2\gamma_{ss} \tag{10i}$$

$$b_{23} = -C_{l_r} k_6 \tag{10j}$$

$$b_{24} = -C_{n_r} k_7 \tag{10k}$$

$$b_{30} = -C_{Y_{\dot\beta}} k_5 \tag{10l}$$

$$b_{31} = -C_{Y_\beta} k_5 \tag{10m}$$

$$b_{32} = -C_{l_{\dot\beta}} k_6 \tag{10n}$$

$$b_{33} = -C_{l_\beta} k_6 \tag{10o}$$

$$b_{34} = -C_{n_{\dot\beta}} k_7 \tag{10p}$$

$$b_{35} = -C_{n_\beta} k_7 \tag{10q}$$

$$b_{42} = -\frac{I_{XZ}}{I_X} \tag{10r}$$

$$b_{43} = -\frac{I_{XZ}}{I_Z} \tag{10s}$$

$$C_{11} = b_{11} + b_{12} \tag{10t}$$

$$C_{21} = 1 + b_{21} \tag{10u}$$

$$C_{30} = 1 + b_{30} \tag{10v}$$

where $k_5 = \frac{\rho S U_{ss}}{2m}$, $k_6 = \frac{\rho S b U_{ss}^2}{2I_X}$, and $k_7 = \frac{\rho S b U_{ss}^2}{2I_Z}$. The trim angle of attack was calculated from

$$\alpha_{tr} = \left\{\frac{2mg}{\rho S U_{ss}^2}\left[(\sigma_T \sin^2 \gamma_{ss} - \sigma_w + \cos \gamma_{ss})\right] - C_{L,o}\right\}\left(C_{L_\alpha}\right)^{-1} \tag{11}$$

Equations (9), (10), and (11) were programmed for the calculator and the program is given in appendix B. The stability derivatives $C_{l_{\dot\beta}}$, $C_{n_{\dot\beta}}$, and $C_{Y_{\dot\beta}}$ have been included in this program. The derivatives C_{l_ϕ} and C_{n_ϕ} are always calculated when wind shears are included. This program calculates all b and C coefficients in the determinant and starts calculating the coefficients of the characteristic equation.

Program 3

Program 3 completes the calculation of the coefficients of the characteristic equation and tests the contents of register 4 to determine if the equation is a quartic or a quintic. If it is a quartic, the number 4 is displayed and the calculator stops. If it is a quintic, the number 5 is displayed and the calculator stops. Label B of this program calculates a real root of the quintic equation by using the secant method. The initial guess for the root is obtained by dividing the coefficient a_4 by 5; this operation is done in the program.

Even with an estimate of the root, however, two estimated points are required to start the secant method. These points are obtained by either adding or subtracting 0.08 from $a_4/5$. The number 0.08 has proved satisfactory for several different fifth-order polynomials. However, the secant method is sensitive to this number and changes may be necessary. Subsequent estimates of the root were calculated from

$$x_{i+1} = x_i - f(x_i)\frac{x_i - x_{i-1}}{f(x_i) - f(x_{i-1})} \tag{12}$$

where x_i is always the present value and $f(x_i)$ is the value of the function being used for $x = x_i$. Synthetic division is used to determine when a root had been found. The fifth-order polynomial is then reduced to a quartic for processing by program 4. During the iteration process, the calculator pauses to display the value of the characteristic polynomial so that convergence can be monitored. When the display shows zero, the root has been found. When the root has been found, the calculator will stop and display 8. The root is in the Y stack register and the time to damp to one-half amplitude or time to double amplitude is in the Z register. A negative number in the Z register means that the value given is the time to double amplitude. The number of iterations required to extract the root is in the T stack register.

The test used for the determination of a root is that the polynomial must be zero to the number of digits in the calculator display; thus, the test for a root assures its accuracy.

Program 4

A quartic equation is the highest order polynomial for which an explicit analytical solution for the root exists. Ferrari's method (refs. 3 and 4) and appendix H was used to obtain the roots of the quartic from the characteristic equation. The general form of the quartic equation is

$$a_4x^4 + a_3x^3 + a_2x^2 + a_1x + a_0 = 0 \tag{13}$$

The first step in applying Ferrari's method is to normalize equation (13) so that $a_4 = 1$. The determination of a real root of the following resolvent cubic is the next step:

$$y^3 + b_2y^2 + b_1y + b_0 = 0 \tag{14}$$

The coefficients of equation (14) are given by

$$\left.\begin{aligned} b_2 &= -a_2 \\ b_1 &= a_1a_3 - 4a_0 \\ b_0 &= a_0\left(4a_2 - a_3^2\right) - a_1^2 \end{aligned}\right\} \tag{15}$$

and the root $Re(y)$ is obtained by

$$Re(y) = S + T = -\frac{b_2}{3} \qquad (f > 0) \tag{16}$$

or

$$Re(y) = 2(R^2 + f)^{1/3} \cos\left[\frac{1}{3}\left(\tan^{-1}\frac{\sqrt{f}}{R}\right)\right] - \frac{b}{2} \qquad (f \leqq 0) \tag{17}$$

where

$$Q = (3b_1 - b_2^2)/9 \tag{18a}$$

$$R = (9b_2b_1 - 27b_0 - 2b_2^3)/54 \tag{18b}$$

$$f = R^2 + Q^3 \tag{18c}$$

$$S = \left(R + \sqrt{f}\right)^{1/3} \quad (18d)$$

$$T = \left(R - \sqrt{f}\right)^{1/3} \quad (18e)$$

The root Re(y) is any root of the resolvent cubic, equation (14); this program is written to calculate the largest real root of equation (14). Once Re(y) is known, the roots of the quartic are obtained by solving the following two quadratic equations:

$$\left.\begin{aligned} z^2 + (A + C)z + (B + D) &= 0 \\ z^2 + (A - C)z + (B - D) &= 0 \end{aligned}\right\} \quad (19)$$

where

$$\left.\begin{aligned} A &= \frac{a_3}{2} \\ B &= \frac{Re(y)}{2} \\ D &= \sqrt{B^2 - a_0} \\ C &= \left(AB - \frac{a_1}{2}\right)\Big/D \qquad (D \neq 0) \\ C &= \sqrt{A^2 - a_2 + Re(y)} \qquad (D = 0) \end{aligned}\right\} \quad (20)$$

Equations (15) to (20) and a quadratic solution routine were programmed to obtain the roots of a quartic equation. The key codes for program 4 are given in appendix D.

Because f and D are tested to determine program direction, special programming is required both to insure that nonsignificant digits do not influence the test and to protect against the small difference of large numbers. The expressions for f and D were written as

$$f = R^2\left(1 + \frac{Q^3}{R^2}\right)$$

$$D = \sqrt{B^2\left(1 - \frac{a_0}{B^2}\right)}$$

for programming. In each case, the quantity in the parenthesis was rounded to the calculator display and then tested. Special routines were added to protect against R and B being equal to 0. The introduction of rounding will introduce some error if a significant number is truncated. As the rounding is controlled by the number of decimal digits in the calculator display, there is flexibility in the amount of rounding introduced. Experience with a set of 20 test equations indicates that a display of 7 digits is satisfactory for most cases.

The roots of the quartic are stored in registers R_1, R_2, S_1, and S_2. The root indicator (-1.0 for complex roots and 0.0 for real roots) is stored in registers R_0 and S_0. If the roots are complex, the real part is stored in register 1 and the imaginary part in register 2.

This program is a general program for the roots of a quartic equation and may be used as a stand-alone program if the coefficients of the quartic are stored in the following locations:

a_3 in register R_0

a_2 in register R_1

a_1 in register R_2

a_0 in register R_3

In addition, this program may be used to solve for the roots of lower order equations. For the cubic where the equation has the form

$$a_3x^3 + a_2x^2 + a_1x + a_0 = 0, \quad a_3 = 1.0$$

the equation is multiplied by x so that it is converted to a quartic with a zero root and the coefficients are stored as follows:

a_2 in register R_0

a_1 in register R_1

a_0 in register R_2

0.0 in register R_3

Quadratic and first-order equations may be solved in a similar manner by multiplying through by x^2 or x^3, respectively.

Program 5

Program 5 calculates the stability parameters (ref. 5, p. 61), such as the time to damp to one-half amplitude or the damping ratio. The equations programmed are given as follows:

Time to damp to one-half amplitude $t_{1/2}$ or time to double amplitude t_D:

$$t_{1/2} \text{ or } t_D = -\frac{0.693}{\text{Re}(\ )} \tag{21}$$

Period:

$$t = \frac{2\pi}{\text{Im}(\ )} \qquad P = \frac{2\pi}{\omega} \tag{22}$$

Number of cycles to damp to one-half amplitude $N_{1/2}$ or time to double amplitude N_D:

$$N_{1/2} \text{ or } N_D = -0.110\,\frac{\text{Im}(\ )}{\text{Re}(\ )} \qquad \text{Re}(\ ) = \sigma,\ \text{Im}(\ ) = \omega \tag{23}$$

Logarithmic decrement:

$$\Delta = \frac{0.693}{N_{1/2} \text{ or } N_D} \tag{24}$$

Undamped circular frequency:

$$\omega_n = \left[(\text{Re}(\ ))^2 + (\text{Im}(\ ))^2\right]^{1/2} \tag{25}$$

Damping ratio:

$$\zeta = \frac{\text{Re}(\ )}{\omega_n} \tag{26}$$

If Δ, t, or N is negative, unstable conditions are indicated. For instance, if $-\frac{0.693}{\text{Re}(\ )}$ is negative, the time calculated is for doubling the amplitude.

The key entries for this program are given in appendix E and the storage at the end of this program contains all the calculated information concerning airplane stability. For real roots, only the time to damp to one-half amplitude or the time to double amplitude is calculated. This program may be used as a stand-alone program.

Program 6

Program 6 uses the polar-rectangular keys of the calculator to implement the Euler transformation used in rigid-body rotation. The transformation programmed is the ψ,θ,ϕ transformation that is frequently used in aeronautics (fig. 1). The use of the polar-rectangular keys permits a short program for this type of transformation.

The transformation scheme is illustrated through the use of a two-dimensional transformation. The coordinates of a point $p(x,y)$ in the xy axis system are given in the $x'y'$ axis system, which is rotated through the angle ε_1 with respect to the xy axis system by

$$\left.\begin{aligned} x' &= x \cos \varepsilon_1 + y \sin \varepsilon_1 \\ y' &= -x \sin \varepsilon_1 + y \cos \varepsilon_1 \end{aligned}\right\} \quad (27)$$

The polar coordinates of $p(x,y)$ are R_*, ε_2 in the xy axis system, where $R_* = (x^2 + y^2)^{1/2}$ and $\varepsilon_2 = \tan^{-1} \frac{y}{x}$, and are R_*, $(\varepsilon_2 - \varepsilon_1)$ in the $x'y'$ axis system. The $x'y'$ axis system coordinates are now given by

$$\left.\begin{aligned} x' &= R_* \cos (\varepsilon_2 - \varepsilon_1) \\ y' &= R_* \sin (\varepsilon_2 - \varepsilon_1) \end{aligned}\right\} \quad (28)$$

If equation (28) is expanded (x is substituted for $R_* \cos \varepsilon_2$ and y is substituted for $R_* \sin \varepsilon_2$), equation (27) results and shows that the same transformation is taking place. This result leads to a program for a two-dimensional transformation. It is assumed that y is stored in the Y stack register, x is stored in the X stack register, and ε_1 is stored in register Rn. The program is as follows:

```
→P

x→y

RCL n

—

x→y

→R
```

This program gives x' and y' in 6 steps instead of the usual 18 steps. This two-dimensional program is completely general. If this two-dimensional transformation program is used in conjunction with a bookkeeping program, three-dimensional transformations may be made. In reference 6 (pp. 272 to 275), a method is given that simplifies the bookkeeping problem. A program for one of the three-dimensional Euler transformations used in aeronautics is given in appendix F. This program is for transformations between two right-hand axes systems (fig. 1) in which the Z-axis is positive downwards. The first rotation is through the angle ψ about the Z_{sp}-axis; the second is through the angle θ about the Y'_{sp}-axis; and the third is through the angle ϕ about the X_b-axis. The angles ψ, θ, and ϕ are the airplane heading, pitch, and roll angles, respectively. The program presented in appendix F is a specialized program because, in three-dimensional transformations, the order in which the rotation angles are taken and the axes about which the rotations take place vary from one transformation to another. Similar programs may be written for other three-dimensional transformations by changing the bookkeeping part of program 6. Subroutines B and C would not be changed.

The advantages of using the polar-rectangular keys in program 6 for three-dimensional transformations are not apparent unless program 6 is compared with a program that uses the traditional approach of calculating the direction cosines and then using them to make the transformation. By using direction cosines, a reasonably efficient program for the ψ,θ,ϕ transformation discussed in this section takes 124 program steps and 20 storage registers, compared with 67 steps and 10 storage registers for the polar-rectangular method of this paper. The impact is even more apparent if both the polar-rectangular (P→R) and the direction-cosine (D-C) methods are considered as subprograms to a main program. Take the following example:

A vector has been computed and its components are stored in three consecutive registers. The angles ψ, θ, and ϕ have also been calculated and are stored in consecutive registers. It is desired to transform the calculated vector components to a new coordinate system rotated from the original by the angles ψ, θ, and ϕ.

Table II summarizes the manner in which the two programs would merge with the main program. Storage for the original vector components and the angles is not counted.

TABLE II.- COMPARISON OF THREE-DIMENSIONAL TRANSFORMATIONS WHEN USED IN PROGRAM

Programming considerations	Space to body		Body to space		Two way	
	P→R method	D-C method	P→R method	D-C method	P→R method	D-C method
Program steps in transformation	26	83	27	92	53	118
Registers used	----	13	----	13	----	13
I register	Used	Used	Used	Used	Used	Used
Storage for new computation	3	3	3	3	6	6
Total program steps used[a]	26	83	27	92	53	118
Total registers used[a]	3	16	3	16	6	19
Steps available for main program	198	141	197	132	171	106
Registers available for main program	22	9	22	9	19	6

[a]I register is not counted.

Analysis of the data presented in table II shows the economics of using the P→R method in programs. In addition, a calculator with only 49 program steps can be programmed one way using the P→R method, and a calculator with 98 steps can handle the two-way P→R transformation. For the D-C method, the smallest programmable calculator that can handle a one-way transformation is one with 98 steps. The two-way transformation will not fit on a 98-step calculator.

USE OF PROGRAMS 1 TO 6

After a program has been keyed into the calculator, the program switch should be set to run. Set the display and trig modes, switch back to program, and record program. The display and trig mode status are now recorded on the magnetic card and the calculator will be set to the indicated status conditions whenever the program is read in. The display and trig mode status are given for each program in the appendixes.

Appendix G contains the check case for the programs in appendixes A to F. To make longitudinal stability calculations, use the following procedure:

(1) Enter program 1 (appendix A)

(2) Enter data as shown on storage map
Push A
At stop, coefficients of characteristics equation have been calculated

(3) Enter program 4 (appendix D)
Push A
At stop, roots of characteristic have been determined

(4) Enter program 5 (appendix E)
Push A
At stop, complete set of longitudinal stability data is stored as indicated on storage map

To make lateral stability calculations, use the following procedure:

(1) Enter program 2 (appendix B)

(2) Enter data as shown on storage map
Push A

(3) At stop, enter program 3 (appendix C)
Push A
If 4 is displayed at stop, go to step 4
If 5 is displayed, push B

When 8 is displayed, the real root, the time to damp to one-half amplitude or the time to double amplitude, and the number of iterations are stored in the stacks

(4) Enter program 4 (appendix D)
Push A
At stop, roots of quartic have been calculated

(5) Enter program 5 (appendix E)
Push A
At stop, a complete set of lateral stability data has been calculated. Data relating to quartic is stored in calculator

Programs 4 and 5 may be used as stand-alone programs.

Program 6 may be used in several different ways. To transform from space axes (X_{sp},Y_{sp},Z_{sp}) to airplane axes (X_b,Y_b,Z_b), use the following procedure:

(1) Enter z_{sp},y_{sp},x_{sp} in stack in order given
Push A

(2) Enter ϕ,θ,ψ in stack in order given
Push B
Push C to make the transformation
At stop, airplane axis coordinates x_b,y_b,z_b are stored in registers R_6, R_7, and R_8, respectively

To transform from airplane axes (X_b,Y_b,Z_b) to space axes (X_{sp},Y_{sp},Z_{sp}), use the following procedure:

(1) Enter z_b,y_b,x_b in stack in order given
Push A

(2) Enter ϕ,θ,ψ in stack in order given
Push B
Push D to make the transformation
At stop, space coordinates x_{sp},y_{sp},z_{sp} are stored in registers R_6, R_7, and R_8, respectively

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APPENDIX A

PROGRAM 1 - LONGITUDINAL AIRPLANE STABILITY

Program 1 uses the basic physical and aerodynamic data of an airplane to calculate the coefficients of the characteristic equation of longitudinal motion. This program calculates normalized coefficients for the characteristic equation. For running this program, the calculator should be set to calculate in radians and to display seven decimal digits. A pause inserted after step ~~49~~ 45 causes the calculated value of α_{tr} to be displayed.

LBLA	Calculate elements of determinant
RCL9	
STO+8	σ_T
RCL0	
x^2	
CHS	$-\bar{c}/k_Y^2$
STO÷(i)	
RCL8	
RCL7	
SIN	
x^2	
×	
RCL9	
−	$\sigma_T \sin^2 \gamma_{ss} - \sigma_w$
STO0	
RCL7	
COS	
STO9	
+	
RCL5	
×	$g(\sigma_T \sin^2 \gamma_{ss} - \sigma_w + \cos \gamma_{ss})$
RCLB	
RCL2	
×	
RCL6	
×	
RCL1	
÷	$\rho S U_{ss}/m$
STOE	
RCL6	
×	
2	
÷	$\rho S U_{ss}^2/2m$
STO×3	
STOB	
÷	C_L
RCL4	
P⇄S	
STO6	C_{m_α} stored in R_6
X⇄Y	
STO5	C_L stored in S_5
RCLD	
−	
RCL1	
÷	α_{tr}
×	
STO+9	
RCL5	
x^2	
RCLC	
÷	
π	
÷	
STO+4	$C_{D,o} + C_L^2/\pi A$
RCL2	
STO+3	$C_{L_{\dot\theta}} + C_{L_{\dot\alpha}}$
RCL8	
STO+7	$C_{m_{\dot\alpha}} + C_{m_{\dot\theta}}$
RCLE	
STO×4	
STO×5	
RCLA	
×	
STO×9	a_{14}
RCLB	
STO×(i)	
STO×0	a_{21}
STO×1	a_{23}
STO×2	
STO×3	a_{22}
RCLA	
STO×6	a_{26}
STO×7	a_{25}
STO×8	a_{35}
P⇄S	
RCL9	
RCL7	
2	
×	
COS	
RCL8	
STO×9	
×	
−	
STOA	
RCL7	
SIN	
STO×9	
RCL9	
2	
×	
−	
RCL5	
STO×(i)	a_{31}
×	
STOB	a_{33}
RCL5	
RCL6	
÷	
STO×9	$g\sigma_T \sin^2 \gamma_{ss}/2U_{ss}$

```
RCL0
×         g(σT sin^2 γss - σw)/Uss
RCL9
RCL3
-
RCL6
P↔S
STO-2     a32
R↓
STO-4     a12
R↓
STO-5     a13
1
0
STOI
RCL3
RCL2
-         a4
STOC
RCL4
×
RCL1
RCLB
-
RCL2
RCL7
×
-
RCL3
RCL8
×
+
STOD
+
STO(i)    a3
ISZ
RCL1
RCL8
×
RCL7
RCL8
×
-
RCL6
RCL2
×
-
STOE
RCLD
RCL4
×
+
RCLA
RCL0
-
RCL5
×
+
STO(i)    a2
ISZ
RCLE
RCL4
×
RCL6
RCLB
×
-
RCL5
RCL7
×
RCL9
RCL3
×
-
RCLA
×
+
RCL5
RCL8
×
RCL9
RCL2
×
-
RCL0
×
-         a1
STO(i)
RCL0
RCL9
×
RCL6
RCL4
×
-
RCLB
×
RCL5
RCL6
×
RCL9
RCL1
×
-
```

```
RCLA
×
+               a0
P↔S
STO3
RCLC
STO÷ 0
STO÷ 1
STO÷ 2
STO÷ 3
RTN
R/S
```

APPENDIX A

Storage Map for Program 1

(i)Address	Register	Input storage	Output storage
0	R_0	k_Y	a_3[a]
1	R_1	m	a_2
2	R_2	ρ	a_1
3	R_3	C_{T_u}	a_0
4	R_4	C_{m_α}	
5	R_5	g	
6	R_6	U_{ss}	
7	R_7	γ_{ss}	
8	R_8	σ_u	
9	R_9	σ_w	
10	S_0	C_{D_α}	a_{21}
11	S_1	C_{L_α}	a_{23}
12	S_2	$C_{L_{\dot{\theta}}}$	a_{32}
13	S_3	$C_{L_{\dot{\alpha}}}$	a_{22}
14	S_4	$C_{D,o}$	a_{12}
15	S_5	0.0	a_{13}
16	S_6	0.0	a_{26}
17	S_7	$C_{m_{\dot{\alpha}}}$	a_{25}
18	S_8	$C_{m_{\dot{\theta}}}$	a_{35}
19	S_9	$C_{m,o}$	a_{14}
20	R_A	$\bar{c}$	a_{31}
21	R_B	S	a_{33}
22	R_C	A	
23	R_D	$C_{L,o}$	
24	R_E	0.0	
25	I	20	

[a]These are the normalized coefficients of the quartic; thus, $a_4 = 1.00$.

APPENDIX B

PROGRAM 2 - LATERAL AIRPLANE STABILITY

Program 2 uses the basic physical and aerodynamic data of an airplane to generate the coefficients of the lateral stability determinant. After completing the calculation of these coefficients, the program starts but does not finish calculating the coefficients of the characteristic equation for lateral motion. For running this program, the calculator should be set to calculate in radians and to display seven decimal digits. A pause inserted after step 44 causes the calculated value of α_{tr} to be displayed.

APPENDIX B

LBLA	
RCL1	
RCL2	
RCL3	
×	
×	
2	
÷	
RCLA	
÷	$\rho S U_{ss}/2m$
CHS	
STO3	
RCL1	
STO÷0	g/U_{ss} in R_0
×	
STO×4	$\rho S b U_{ss}^2/2m$
RCL4	
STOE	
RCL5	
x^2	
STO5	
STO÷4	$\rho S b U_{ss}^2/2I_X$
RCL9	
x^2	
STO9	
STO÷(i)	$\rho S b U_{ss}^2/2I_Z$
RCL8	
X<0	
SF2	
x^2	
F?2	
CHS	
CHS	
ENT	
ENT	
RCL9	
÷	I_{XZ}/I_Z
STO9	
X→Y	
RCL5	
÷	
STO8	I_{XZ}/I_X
RCL0	
RCL7	
RCL1	
÷	
×	
STO2	
RCLC	
COS	
RCL0	
×	
−	$\frac{g\sigma_w}{U_{ss}^2} - \frac{g}{U_{ss}} \cos \gamma_{ss}$
RCL2	
RCL0	
RCL1	
÷	
RCL6	
STO×0	
×	
+	
2	
÷	
RCLC	
2	
×	
sin	
×	
STOA	
R↓	
STOC	
RCL3	
GSBa	b_{12}, C_{21}
DSZ	
GSBa	b_{21}
1	
STO+(i)	
X→Y	
DSZ	
GSBa	b_{30}
X→Y	
STO+(i)	C_{30}
X→Y	
GSBa	
RCL4	
GSBa	b_{32}
GSBa	b_{23}
RCL(i)	
STO6	
R↓	
GSBa	b_{14}
GSBa	b_{33}
RCLE	
GSBa	b_{34}
GSBa	b_{16}
GSBa	b_{35}
GSBa	b_{24}
RCL(i)	
STO7	

RCL0	
STO×6	
STO×7	
2	
3	
STOI	
RCL8	Calculate coefficient of characteristic equation
P↔S	Secondary called
RCL3	
×	
RCL7	
-	$b_{42}b_{34} - b_{32}$
STOE	
RCL6	
RCL1	
×	
RCL0	
RCL4	
×	
-	$b_{23}b_{35} - b_{24}b_{33}$
RCL6	
RCL3	
×	
RCL0	
RCL7	
×	
-	
RCL4	
-	
RCL1	
P↔S	Primary called
RCL8	
×	
+	$b_{23}b_{34} - b_{24}b_{32} - b_{33} + b_{42}b_{35}$
GSBb	Complete calculations and store
STO2	
R↓	
STO1	
R↓	
GSBb	
STO3	
R↓	
STO+2	
RCLE	
GSBb	
STO+1	
R↓	
STO0	
RCL9	
P↔S	Secondary called
RCL7	
×	
CHS	
RCL3	
+	$b_{34} - a_{32}a_{34}$
RCL1	
RCL5	
RCL3	
×	
+	
RCL2	
RCL7	
×	
-	
RCL4	
P↔S	Primary called
RCL9	
×	
-	$b_{35} + b_{34}b_{14} - b_{16}b_{32} - b_{33}b_{43}$
DSZ	
DSZ	
GSBb	Complete calculations and store terms
STO-2	
R↓	
STO-1	
R↓	
GSBb	
STO-1	
R↓	
STO-0	
RTN	End of program
LBLa	Subroutines for calculation of coefficients
DSZ	
STO×(i)	
RTN	
LBLb	
ENT↑	
ENT↑	
RCL(i)	
×	
X↔Y	
DSZ	
RCL(i)	
×	
ISZ	
RTN	
R/S	

APPENDIX B

Storage Map for Program 2

(i)Address	Register	Initial storage	End of program
0	R_0	g	(a)
1	R_1	U_{ss}	(a)
2	R_2	ρ	(a)
3	R_3	S	(a)
4	R_4	b	(a)
5	R_5	k_X	
6	R_6	σ_u	b_{15}
7	R_7	σ_w	b_{17}
8	R_8	[b]k_{XZ}	b_{42}
9	R_9	k_Z	b_{43}
10	S_0	C_{n_r}	b_{24}
11	S_1	$C_{n\beta}$	b_{35}
12	S_2	C_{n_p}	b_{16}
13	S_3	$C_{n\dot{\beta}}$	b_{34}
14	S_4	$C_{l\beta}$	b_{33}
15	S_5	C_{l_p}	b_{14}
16	S_6	C_{l_r}	b_{23}
17	S_7	$C_{l\dot{\beta}}$	b_{32}
18	S_8	$C_{Y\beta}$	b_{31}
19	S_9	$C_{Y\dot{\beta}}$	C_{30}
20	R_A	m	b_{22}
21	R_B	C_{Y_r}	C_{21}
22	R_C	γ_{ss}	b_{13}
23	R_D	C_{Y_p}	C_{11}
24	R_E	0.0	
25	I	24	

[a]Registers R_0 to R_4 contain the partially calculated coefficients of the characteristic equation.

[b]If k_{XZ} is imaginary, enter k_{XZ} as a negative number.

PROGRAM 3 - LATERAL AIRPLANE STABILITY (Concluded)

Program 3 completes the calculation of the coefficients of the characteristic equation of lateral motion that was started in program 2. The program then determines if the characteristic equation is a quartic or a quintic. If it is a quartic, a 4 is displayed and the program stops. Program 4 is then used to obtain the roots of the quartic. If the characteristic equation is a quintic, a 5 is displayed and the program continues on to extract the real root of the quintic and then calculates the time to damp to one-half amplitude or the time to double amplitude. This program uses the storage that existed at the end of program 2. This program calculates normalized coefficients for the quartic and the quintic. For running this program, the calculator should be set to calculate in radians and to display seven decimal digits.

LBLA	
RCL7	
RCL6	
P⇄S	Secondary called
RCL3	
×	
X⇄Y	
RCL7	
×	
-	
RCL2	
RCL4	
×	
-	
RCL5	
RCL1	
×	
+	$b_{15}b_{34} - b_{32}b_{17}$
RCL4	$- b_{16}b_{33} + b_{14}b_{35}$
RCL1	
P⇄S	Primary called
RCL6	
×	
X⇄Y	
RCL7	
×	
-	$b_{35}b_{15} - b_{33}b_{17}$
GSBa	Complete and store
CHS	calculations
STO4	
R↓	
STO-3	
R↓	
GSBa	
STO-3	
R↓	
STO-2	
RCL8	
RCL9	
P⇄S	Secondary called
RCL6	
×	
CHS	
X⇄Y	
RCL2	
×	
-	
RCL5	
+	$b_{14} + b_{24} - b_{16}b_{42}$
RCL0	$- b_{43}b_{23}$
+	
RCL6	
RCL0	
P⇄S	Primary called
RCL6	
×	
X⇄Y	
RCL7	
×	
-	$b_{24}b_{15} - b_{23}b_{17}$
DSZ	
DSZ	
GSBa	
STO+3	
R↓	
STO+2	
R↓	
GSBa	
STO+1	
R↓	
STO0	
1	
RCL8	
RCL9	
×	
-	$1 - b_{42}b_{43}$
RCL6	
RCL7	
RCL8	
×	
-	
P⇄S	Secondary called
RCL2	
RCL6	
×	
-	
RCL5	
RCL0	
×	
+	$b_{15} - b_{17}b_{42}$
	$- b_{16}b_{23} + b_{14}b_{24}$
P⇄S	Primary called
GSBa	Complete and store
STO+2	calculations
R↓	
STO+1	
R↓	
GSBa	
STO+0	
R↓	
STOE	
RCL4	Determines if equa-
X≠0	tion is a quartic
GOTO1	or a quintic

```
GSBd
4         Indicates quartic
RTN       Stop for quartic
LBL1
GSBd
5         Indicates quintic
RTN       Stop for quintic
LBLB      Calculate real root
RCL0        of quintic
STOA
RCL1      This section
STOB        positions data
RCL2
STOC
RCL3
STOD
RCL4
STOE
0         Initialization for
STO7        secant method
FIX
RCLA
5
÷
.
0
8
-
STO5
STO6
GSBb
STO8
.
1
6
STO+6
LBL0      Evaluates polynomial
1           and tests for
STO+7       solution
GSBb
STO9
RND
Pause     Displays value of
X=0         polynomial
GOTO1       polynomial
RCL6      Calculates and stores
RCL5        new value of  X
RCL6
STO5
X→Y
-
```

```
RCL8
RCL9
STO8
X→Y
-
÷
RCL8
×
-
STO6
GOTO0
LBL1      Output routine
RCL7
.
6
9
3
CHS
RCL6
÷
RCL6
8
RTN       End of program
LBLa      Subroutine for
ENT↑        calculating
ENT↑        coefficient of
RCL(i)      characteristic
×           equation
X→Y
DSZ
RCL(i)
×
ISZ
RTN
LBLb      Polynomial evalu-
2           ation subroutine
0
STOI
1
GSBc
STO0
GSBc
STO1
GSBc
STO2
GSBc
STO3
GSBc
STO4
RTN
LBLc      Synthetic division
            subroutine
```

```
RCL6
×
RCL(i)
+
ISZ
RTN
LBLd        Normalization
RCLE          subroutine
STO÷0
STO÷1
STO÷2
STO÷3
STO÷4
RTN
R/S
```

APPENDIX C

Storage Map for Program 3

(i)Address	Register	Initial storage[a]	End of program[b]
0	R_0		a_3
1	R_1		a_2
2	R_2		a_1
3	R_3		a_0
4	R_4		
5	R_5		
6	R_6	b_{15}	
7	R_7	b_{17}	
8	R_8	b_{42}	
9	R_9	b_{43}	
10	S_0	b_{24}	
11	S_1	b_{35}	
12	S_2	b_{16}	
13	S_3	b_{34}	
14	S_4	b_{33}	
15	S_5	b_{14}	
16	S_6	b_{23}	
17	S_7	b_{32}	
18	S_8	b_{31}	
19	S_9	C_{30}	
20	R_A	b_{22}	a_4 Coefficients of quintic
21	R_B	C_{21}	a_3 Coefficients of quintic
22	R_C	b_{13}	a_2 Coefficients of quintic
23	R_D	C_{11}	a_1 Coefficients of quintic
24	R_E		a_0 Coefficients of quintic
25	I	in use	

[a]The initial storage is the same as that at end of program 2. The partially calculated coefficients of the characteristic equation are stored in R_0 to R_4.

[b]The end storage is the same for display signals 4 and 8; the normalized coefficients of the quartic are in registers R_0 to R_3. The real root of the quintic is in the Y register and the time to damp to one-half amplitude or the time to double amplitude is in the Z register when 8 is displayed. Pressing R↓ moves the real root of the quintic to the X register; pressing R↓ again moves the time to damp to one-half amplitude or the time to double amplitude to the X register. The number of iterations required to obtain the root is in the stack T register and may be obtained by pressing R↓.

APPENDIX D

PROGRAM 4 - ROOTS OF A QUARTIC EQUATION

Program 4 applies Ferrari's method for the roots of a quartic equation to the output of either program 1 or program 3 to determine the remaining eigenvalues of the characteristic equation of longitudinal or lateral motion. Normalized coefficients must be used for this program. For running this program, the calculator should be set to calculate in radians and to display seven decimal digits.

APPENDIX D

LBLA	Calculate coefficients of resolvent cubic
4	
STO6	
RCL1	
STOC	
STO×6	
CHS	
STO4	b_2 in R_4
RCL0	
STOB	
STO5	
x^2	
STO−6	
RCL2	
STOD	
STO×5	
x^2	
RCL3	
STOE	
STO×6	
4	
×	
STO−5	b_1 in R_5
X↔Y	
STO−6	b_0 in R_6
3	Calculate Q, R, Q^3, R^2, and f
STO÷4	
STO÷5	
RCL5	
RCL4	
x^2	
−	Q
X↔Y	
STO×5	
Y^x	Q^3
RCL4	
RCL5	
×	
RCL6	
−	
2	
÷	
RCL4	
3	
Y^x	
−	R
STO7	
x^2	R^2
STOA	
X≠0	
GOTO0	
GSBa	
GOTO1	
LBL0	
÷	
1	
GSBa	
RCLA	
×	
LBL1	f
ABS	Calculate largest real root of resolvent cubic
$\sqrt{\ }$	
F?2	
GOTO2	
RCL7	
→P	
GSBb	
2	
×	
X↔Y	
3	
÷	
COS	
×	
GOTO3	
LBL2	
RCL7	
X↔Y	
STO−7	
+	
GSBb	
RCL7	
GSBb	
+	
LBL3	
RCL4	
−	Re(y)
RND	
Pause	Display Re(y)
STO8	Calculate A, B, A ± C, B ± D, C, D
STO9	
2	
STO÷0	A
STO÷2	
STO÷8	B
RCL0	
STO6	
RCL8	
STO7	
×	
RCL2	
−	C, D ≠ 0
1	
RCL3	

```
CHS
RCL8
X^2
X≠0
GOTO4
X↔Y
GSBa
GOTO5
LBL4
÷
GSBa
RCL8
X^2
×
LBL5
√          D
STO+7      B + D
STO-8      B - D
F?2
GOTO6
RCL0
X^2
RCL1
-
RCL9
+
√          C,  D = 0
GOTO7
LBL6
÷
LBL7
STO+0      A + C
STO-6      A - C
RCL7       Solve for roots
RCL0         of quartic
GSBc
RCL8
RCL6
P↔S
GSBc
P↔S
RTN
LBLa       Subroutine used in
+            calculation of  f
RND          and  D
Pause      Displays quantity
X>0          tested
SF2
RTN
LBLb       Subroutine for cube
X<0          root of positive or
SF2          negative number
```

```
ABS
3
1/X
Y^X
F?2
CHS
RTN
LBLc       Guadratic solution
X≠0          subroutine
GOTO8      Protects against
X↔Y          case
X=0          A ± C = B ± D = 0
GOTO9
X↔Y
LBL8
2
÷          Calculates  -(b/2)
CHS          and  (b/2)^2 - c
STO4
X^2        Determines if roots
X↔Y          are real or
STO5         complex
-
X<0
GOTO0
√          Solves for real
RCL4         roots
X<0
SF2
X↔Y
F?2
CHS
+
STO÷5
RCL5
X↔Y
0
GOTO1
LBL0       Solves for complex
ABS          roots
√
RCL4
1
CHS
GOTO1
LBL9       Enters zero roots
ENT↑
ENT↑
LBL1       Stores and displays
STO0         roots
-X-
R↓
```

APPENDIX D

```
STO1
-X-
R↓
STO2
-X-
RTN
```

APPENDIX D

Storage Map for Program 4

(i) Address	Register	Initial storage[a]	End of program[b]
0	R_0	a_3	Root-type indicator
1	R_1	a_2	Re( ) or α_1
2	R_2	a_1	Im( ) or α_2
3	R_3	a_0	
4	R_4		
5	R_5		
6	R_6		
7	R_7		
8	R_8		
9	R_9		
10	S_0		Root-type indicator
11	S_1		Re_1( ) or α_3
12	S_2		Im_1( ) or α_4
13	S_3		
14	S_4		
15	S_5		
16	S_6		
17	S_7		
18	S_8		
19	S_9		
20	R_A		
21	R_B		a_3 } Coefficients of quartic
22	R_C		a_2 } Coefficients of quartic
23	R_D		a_1 } Coefficients of quartic
24	R_E		a_0 } Coefficients of quartic
25	I		

[a]Initial storage is provided by output of program 1 or program 3.

[b]The root-type indicator is 0 for real roots and −1 for complex roots. The real part of the complex root is stored in R_1 or S_1 and the imaginary part in R_2 or S_2.

APPENDIX E

PROGRAM 5 - STABILITY PARAMETERS

Program 5 utilizes the eigenvalues computed by program 4 to calculate stability parameters, such as the time to damp to one-half amplitude or the damping ratio. For running this program, the calculator should be set to calculate in radians and to display seven decimal digits.

APPENDIX E

Step	Comment
LBLA	Clears registers and protects roots and root indicators
RCL0	
RCL1	
RCL2	
CLRREG	
STO2	
R↓	
STO1	
R↓	
STO0	
P⇌S	
RCL0	
RCL1	
RCL2	
CLRREG	
STO2	
R↓	
STO1	
R↓	
STO0	
P⇌S	
.	Stores constants and initializes I register
6	
9	
3	
CHS	
STOA	
.	
1	
1	
CHS	
STOB	
π	
2	
STOI	
×	
STOC	
LBLB	Determines if roots are real or complex
RCL0	
X≠0	
SF2	
RCL1	Protects against zero roots
ABS	
RCL2	
ABS	
+	
X=0	
GOTO1	
F?2	Switch for complex roots
GOTO2	
RCLA	Calculates $t_{1/2}$ or t_D for real roots
STO3	
STO4	
RCL1	
STO÷3	
RCL2	
STO÷4	
GOTO1	
LBL2	Calculate stability parameters for complex roots
RCL1	Protects against zero real part of complex root
X=0	
GOTO3	
RCLA	
STO3	
STO4	
X⇌Y	
STO÷3	$t_{1/2}$ or t_D
STO8	
RCL2	
X⇌Y	
÷	
RCLB	
×	$N_{1/2}$ or N_D
STO5	
CHS	
STO÷4	Δ
LBL3	
RCLC	
STO6	
RCL2	
STO÷6	t
RCL1	
→P	
STO7	ω_n
CHS	
STO÷8	ζ
LBL1	
P⇌S	Switch for second set of roots and program stop
DSZ	
GOTOB	
RTN	
R/S	

APPENDIX E

Storage Map for Program 5

(i)Address	Register	Initial storage[a]	End of program
0	R_0	Root-type indicator	Root-type indicator
1	R_1	Re( ) or α_1	Re( ) or α_1
2	R_2	Im( ) or α_2	Im( ) or α_2
3	R_3		$t_{1/2}$ or t_D
4	R_4		Δ
5	R_5		$N_{1/2}$ or N_D
6	R_6		t
7	R_7		ω_n
8	R_8		ζ
9	R_9		
10	S_0	Root-type indicator	Root-type indicator
11	S_1	Re_1( ) or α_3	Re_1( ) or α_3
12	S_2	Im_1( ) or α_4	Im_1( ) or α_4
13	S_3		$t_{1/2}$ or t_D
14	S_4		Δ
15	S_5		$N_{1/2}$ or N_D
16	S_6		t
17	S_7		ω_n
18	S_8		ζ
19	S_9		
20	R_A		
21	R_B		
22	R_C		
23	R_D		
24	R_E		
25	I		

[a]The initial storage is the same as the storage at the end of program 4.

For real roots, only the time to damp to one-half amplitude or the time to double amplitude is calculated. This quantity is stored in register 3 for the root in register 1 and in register 4 for the root in register 2.

PROGRAM 6 - EULER TRANSFORMATION FOR AERONAUTICS

Program 6 is for the standard Euler transformation that is used in aeronautics between inertial axes and airplane axes. The trigonometric mode and the number of decimal digits in the display are assigned by the user.

APPENDIX F

```
LBLA       Stores Xsp,Ysp,Zsp
STO0         or Xb,Yb,Zb
R↓
STO1
R↓
STO2
RTN
LBLB       Stores ψ, θ, φ
STO3
R↓
STO4
R↓
STO5
RTN
LBLC       Transforms Xsp,
3            Ysp,Zsp to
STOI         Xb,Yb,Zb
RCL1
RCL0
GSBb
RCL2
GSBb
X↔Y
STO6
R↓
X↔Y
GSBb
STO7
R↓
STO8
RTN
LBLb       Transformation sub-
→P           routine Xsp,Ysp,
X↔Y          Zsp to Xb,Yb,Zb
RCL(i)
-
X↔Y
→R
ISZ
RTN
LBLD       Transforms Xb,Yb,Zb
5            to Xsp,Ysp,Zsp
STOI
RCL2
RCL1
GSBc
X↔Y
RCL0
X↔Y
GSBc
STO8
R↓
GSBc
STO6
R↓
STO7
RTN
LBLc       Transformation
→P           subroutine Xb,
X↔Y          Yb,Zb to Xsp,
RCL(i)       Ysp,Zsp
CHS
-
X↔Y
→R
DSZ
RTN
R/S
```

APPENDIX G

CHECK CASES FOR PROGRAMS 1 TO 6

This appendix gives check cases for each program given in appendixes A to F. Each check case is complete in itself and does not depend on the output of a previous program. For program 3, two check cases are given - one for label A and one for label B. There is no check case given for programs 1, 2, and 3 for $\sigma_u = \sigma_w = 0.0$. All check cases are independent of previous results.

APPENDIX G

Check Case for Program 1

Register	Input storage	Output
R_0	$k_Y = 10.463784$	$a_3 = 1.3924836$
R_1	$m = 90909.1$	$a_2 = 1.1016636$
R_2	$\rho = 1.2929$	$a_1 = -0.0160353$
R_3	$C_{T_u} = -0.000248411$	$a_0 = -0.0210558$
R_4	$C_{m_\alpha} = -1.115$	
R_5	$g = 9.80665$	
R_6	$U_{ss} = 77.12$	
R_7	$\gamma_{ss} = -0.052359878$	
R_8	$\sigma_u = 2.0$	
R_9	$\sigma_w = 0.0$	
S_0	$C_{D_\alpha} = 0.529$	$a_{21} = 5.9757330$
S_1	$C_{L_\alpha} = 4.87$	$a_{23} = 55.0128915$
S_2	$C_{L_{\dot\theta}} = 0.283$	$a_{32} = -73.9231523$
S_3	$C_{L_{\dot\alpha}} = 0.0889$	$a_{22} = 4.2010871$
S_4	$C_{D,o} = 0.038$	$a_{12} = 0.0316972$
S_5	0.0	$a_{13} = 0.2546699$
S_6	0.0	$a_{26} = 0.8064467$
S_7	$C_{m_{\dot\alpha}} = -0.241$	$a_{25} = 0.6856605$
S_8	$C_{m_{\dot\theta}} = -0.707$	$a_{35} = 0.5113523$
S_9	$C_{m,o} = 0.0$	$a_{14} = 0.0007159$
R_A	$\bar{c} = 7.0104$	$a_{31} = -9.7126460$
R_B	$S = 267.1$	$a_{33} = 1.5369077$
R_C	$A = 7.03$	$a_4 = 78.1242394$
R_D	$C_{L,o} = 0.705$	
R_E	0.0	
I	20	

APPENDIX G

Check Case for Program 2

Register	Input storage	Output
R_0	$g = 9.80665$	0.0
R_1	$U_{ss} = 77.12$	0.6027688
R_2	$\rho = 1.2929$	0.3089144
R_3	$S = 267.1$	0.0064130
R_4	$b = 43.4$	-11.394069
R_5	$k_X = 6.559296$	
R_6	$\sigma_u = 2.0$	$b_{15} = -0.1779356$
R_7	$\sigma_w = -0.5$	$b_{17} = 0.0473607$
R_8	$k_{XZ} = -1.28016$	$b_{42} = 0.0380903$
R_9	$k_Z = 12.249912$	$b_{43} = 0.0109210$
S_0	$C_{n_r} = -0.057$	$b_{24} = 0.1862234$
S_1	$C_{n_\beta} = 0.173$	$b_{35} = -0.5652042$
S_2	$C_{n_p} = -0.0182$	$b_{16} = 0.0594608$
S_3	$C_{n_{\dot{\beta}}} = 0.0$	$b_{34} = 0.0$
S_4	$C_{l_\beta} = -0.21$	$b_{33} = 2.3929305$
S_5	$C_{l_p} = -0.111$	$b_{14} = 1.2648347$
S_6	$C_{l_r} = 0.0614$	$b_{23} = -0.6996473$
S_7	$C_{l_{\dot{\beta}}} = 0.0$	$b_{32} = 0.0$
S_8	$C_{Y_\beta} = -0.866$	$b_{31} = 0.1268488$
S_9	$C_{Y_{\dot{\beta}}} = 0.0$	$C_{30} = 1.0$
R_A	$m = 90909.1$	$B_{22} = -0.0001293$
R_B	$C_{Y_r} = 0.0881$	$C_{21} = 0.9870954$
R_C	$\gamma_{ss} = -0.052359878$	$b_{13} = -0.1278111$
R_D	$C_{Y_p} = 0.0539$	$C_{11} = -0.0078951$
R_E	0.0	
I	24	21

APPENDIX G

Check Case for Program 3 - Label A

Register	Input storage	Output[a]
R_0	0.0	a_4 = 1.5838890
R_1	0.6854141	a_3 = 0.9679675
R_2	0.3031611	a_2 = 1.1621140
R_3	0.0063915	a_1 = 0.0095553
R_4	-490.2586228	a_0 = -0.0001405
R_5		
R_6	b_{15} = -0.1779356	
R_7	b_{17} = 0.0473607	
R_8	b_{42} = 0.0380903	
R_9	b_{43} = 0.0109210	
S_0	b_{24} = 0.1862234	
S_1	b_{35} = -0.5652042	
S_2	b_{16} = 0.0594608	
S_3	b_{34} = 0.0	
S_4	b_{33} = 2.3929305	
S_5	b_{14} = 1.2648347	
S_6	b_{23} = -0.6996473	
S_7	b_{32} = 0.0	
S_8	b_{31} = 0.1268488	
S_9	C_{30} = 1.0	
R_A	b_{22} = -0.0110087	
R_B	C_{21} = 0.9870954	
R_C	b_{13} = -0.1273814	
R_D	C_{11} = -0.0421244	
R_E		
I	21	

[a]They are normalized coefficients for the quintic; thus, a_5 = 1.0.

APPENDIX G

Check Case for Program 3 - Label B

Register	Input	Output (coefficient of quartic)
R_0	a_4 = 1.583889	a_3 = 1.5915011
R_1	a_3 = 0.9679675	a_2 = 0.9800821
R_2	a_2 = 1.1621140	a_1 = 1.1695744
R_3	a_1 = 0.0095552	a_0 = 0.0184581
R_4	a_0 = -0.0001405	
R_5		
R_6		
R_7		
R_8		
R_9		
S_0		
S_1		
S_2		
S_3		
S_4		
S_5		
S_6		
S_7		
S_8		
S_9		
R_A		
R_B		
R_C		
R_D		
R_E		
I		

The stack contains the quintic data as follows:

Stack register	T	Number of iterations	12
Stack register	Z	t_D or $t_{1/2}$	t_D = 91.0398496 (displayed as negative number)
Stack register	Y	Root	0.0076121
Stack register	X	8.0	Indicates root has been found

Use the R↓ to move data into the X register for recording.

APPENDIX G

Check Case for Program 4

Store:

a_3 = 1.4007102 in R_0
a_2 = 1.1058038 in R_1
a_1 = -0.0158317 in R_2
a_0 = -0.0227494 in R_3

Results:

R_0	Root indicator	-1.00 (indicates complex roots)
R_1	Real part	-0.6946683
R_2	Imaginary part	0.7924165
S_0	Root indicator	0.00 (indicates real roots)
S_1	First real root	-0.1489289
S_2	Second real root	0.1375553

APPENDIX G

Check Case for Program 5

Register	Input	Output	
R_0	-1.0	-1.0	Root
R_1	-0.6946683	-0.6946683	indicator
R_2	0.7924165	0.7924165	and roots
R_3		$t_{1/2}$ = 0.9975984	
R_4		Δ = 5.5228662	
R_5		$N_{1/2}$ = 0.1254783	
R_6		t = 7.9291450	
R_7		ω_n = 1.0537969	
R_8		ζ = 0.6592051	
R_9			
S_0	0.0	0.0	Root
S_1	-0.1489288	-0.1489288	indicator
S_2	0.1375553	0.1375553	and roots
S_3		$t_{1/2}$ = 4.6532303	First root
S_4		t_D = -5.0379738	Second root
S_5			
S_6			
S_7			
S_8			
S_9			
R_A			
R_B			
R_C			
R_D			
R_E			
I			

APPENDIX G

Check Case for Program 6

Space axes (X_{sp}, Y_{sp}, Z_{sp}) to body axes (X_b, Y_b, Z_b):

$x_{sp} = y_{sp} = z_{sp} = 1.0$ $\psi = 25^{\circ}$; $\theta = 10^{\circ}$; $\phi = 30^{\circ}$

Results:

$x_b = 1.1351$ in R_6
$y_b = 1.0267$ in R_7
$z_b = 0.8109$ in R_8

Body axes (X_b, Y_b, Z_b) to space axes (X_{sp}, Y_{sp}, Z_{sp}):

$x_b = 1.1351$ $y_b = 1.0267$ $z_b = 0.8109$ $\psi = 25^{\circ}$; $\theta = 10^{\circ}$; $\phi = 30^{\circ}$

Results:

$x_{sp} = 1.0000$ in R_6
$y_{sp} = 1.0000$ in R_7
$z_{sp} = 1.0000$ in R_8

APPENDIX H

A DISCUSSION OF FERRARI'S METHOD FOR THE SOLUTION OF A QUARTIC EQUATION

Ferrari (1522-1575), an Italian mathematician, obtained the solution of a quartic by reducing the problem to the solution of two quadratic equations. As the details of obtaining the quadratic equations are not consistent among authors, the details of obtaining the quadratics used for the solution in this paper are presented.

The general quartic equation is

$$x^4 + a_3x^3 + a_2x^2 + a_1x + a_0 = 0 \tag{H1}$$

Rewrite this equation as

$$x^4 + a_3x^3 = -a_2x^2 - a_1x - a_0 \tag{H2}$$

and complete the square

$$\left(x^2 + \frac{a_3}{2}x\right)^2 = \left(\frac{a_3{}^2}{4} - a_2\right)x^2 - a_1x - a_0 \tag{H3}$$

Now, add $\left(x^2 + \frac{a_3}{2}x\right)y + \frac{y^2}{4}$ to each side of equation (H3), y being a dummy variable

$$\left(x^2 + \frac{a_3}{2}x + \frac{y}{2}\right)^2 = \left(\frac{a_3{}^2}{4} - a_2 + y\right)x^2 + \left(\frac{a_3}{2}y - a_1\right)x + \left(\frac{y^2}{4} - a_0\right) \tag{H4}$$

The left-hand side of equation (H4) is a perfect square. If the right-hand side is also a perfect square, it can be written as the square of a linear function of x, say $Cx + D$. Thus, the pair of quadratics that must be solved for the roots of the quartic are

$$x^2 + \frac{a_3}{2}x + \frac{y}{2} = \pm(Cx + D) \tag{H5}$$

The right-hand side of equation (H4) is a perfect square if, and only if, its discriminant is 0

$$\left(\frac{a_3 y}{4} - \frac{a_1}{2}\right)^2 - \left(\frac{a_3^2}{4} - a_2 + y\right)\left(\frac{y^2}{4} - a_0\right) = 0 \qquad \text{(H6)}$$

In this equation y has not been defined, and if equation (H6) is written as a function of y, it becomes

$$y^3 - a_2 y^2 + (a_3 a_1 - 4a_0)y + \left[a_0(4a_2 - a_3^2) - a_1^2\right] = 0 \qquad \text{(H7)}$$

This equation is called the resolvent cubic and any root y_i of equation (H7) insures that equation (H6) is 0.

All that remains is the determination of the coefficients C and D. The discriminant equation (H6)

$$\left(\frac{a^2}{4} - a_2 + y\right) = \left(\frac{a_3 y}{4} - \frac{a_1}{2}\right)^2 \Bigg/ \left(\frac{y^2}{4} - a_0\right)$$

permits the right-hand side of equation (H4) to be written as

$$\frac{\left(\frac{a_3 y}{4} - \frac{a_1}{2}\right)^2}{\frac{y^2}{4} - a_0} x^2 + \left(\frac{a_3 y}{2} - a_1\right)x + \left(\frac{y^2}{4} - a_0\right)$$

which is a perfect square, and the coefficients C and D are

$$C = \left(\frac{a_3 y}{4} - \frac{a_1}{2}\right) \Bigg/ \sqrt{\frac{y^2}{4} - a_0} \qquad \text{(H8)}$$

$$D = \sqrt{\frac{y^2}{4} - a_0} \qquad \text{(H9)}$$

only if $D \neq 0$. The right-hand side of equation (H4) as written is a perfect square because

$$\frac{a_3 y}{2} - a_1 = 2\sqrt{\left(\frac{a_3^2}{4} - a_2 + y\right)\left(\frac{y^2}{4} - a_0\right)}$$

so that

$$C = \sqrt{\frac{a_3^2}{4} - a_2 + y} \qquad \text{(H10)}$$

and are used in place of equation (H8) if $D = 0$.

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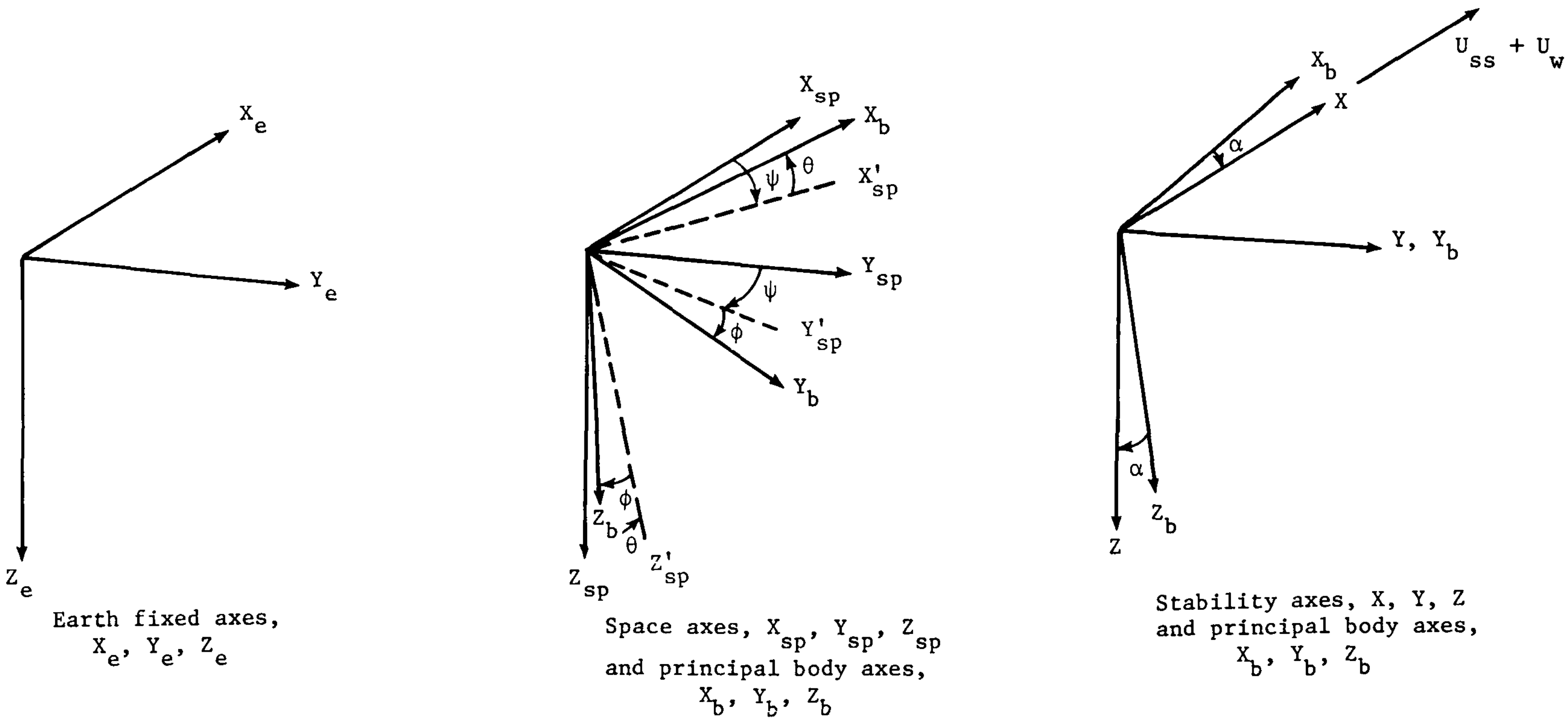

Figure 1.- Coordinate systems and Euler angles. Order of rotation for Euler angles is ψ, θ, and ϕ. Moving axes translate with airplane and remain parallel to Earth fixed axes. Positive directions are shown.

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16 Abstract

Programs are presented for calculating airplane stability characteristics with a card programmable pocket calculator. These calculations include eigenvalues of the characteristic equations of lateral and longitudinal motion as well as stability parameters such as the time to damp to one-half amplitude or the damping ratio. The effects of wind shear are included. Background information and the equations programmed are given. The programs are written for the International System of Units, the dimensional form of the stability derivatives, and stability axes. In addition to the programs for stability calculations, an unusual and short program is included for the Euler transformation of coordinates used in airplane motions. The programs have been written for a Hewlett Packard HP-67 calculator. However, the use of this calculator does not constitute an endorsement of the product by the National Aeronautics and Space Administration.

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