Editor Gene Hegedus can be reached at P.O. Box 2151, Oxnard, CA 93034. Publication price is $2.65 per issue?

The following four books were seen on the shelves of Data Domain in Schaumburg, Illinois during a Chicago Chapter meeting hosted by Data Domain. (312) 397-8700

Solving Business Problems on the Electronic Calculator is an 8.5" x 5.6" soft spiral bound "how to do" book by James Meehan and Allan Doerr. This 217 page workbook like book is available from the Gregg and Community College Division of McGraw-Hill Book Company in the $7 price range. The authors acknowledge Victor Comptometer Corporation and Sharp Electronics Corporation in this 1975 Copyrighted book.

Arithmetic & calculators is a book covering "How to Deal with Arithmetic in the Calculator Age". It is available from W.H. Freeman and Company. Authored by William G. Chinn, Richard, A. Dean and Theodore N. Tracewell the price and copyright of this 9.2" x 6.6" paperback is unknown.

How to Get the Most Out of Your Low-Cost Electronic Calculator is a Hayden Book by Ronald M. Benrey. This 9" x 6" paperback is in the $5 price range, Hayden No. 5942.

Basic Electronics Math with a Scientific Calculator by Edward M. Noll is an 8.4" x 5.4" Howard W. Sams & Co. Inc. paperback No. 21425.

Another new magazine/Journal. *Didactic Programming* , is published by Educational Calculator Devices and edited by Professor Arthur David Snider. A "Journal of Calculator-Demonstrated Math Instruction (CDMI)", *Diadactic Programming,* is currently free to instructors of mathematics who have requested it. If you are interested write on your school's letterhead to the address below. Articles in the first - Fall, 1978 - issue include:

"Some Instructive Calculator Demonstrations of Iterative Equation Solvers"

"Fibonacci Search"

"SIMPLEX - Gauss Elimination on the Calculator"

"SR-56 - 3 Equations in 3 Unknowns"

This first issue had most of the programs with code for several HP and TI machines. Additional copies of the first issue are $1 each to cover postage and handling.

Didactic Programming, P.O. Box 974, Laguna Beach, CA 92652 R/S

SUCCESSIVE BISECTIONS

SUCCESSIVE BISECTIONS A SIMPLE ALTERNATIVE?

There are several methods that can be used to solve equations of the form $f(x)=0$ where $f(x)$ represents a general function. Such problems arise naturally in any situation in which there is an equation relating several variables, where the values of all but one of the variables are known and the value of the remaining variable is to be found.

For example, in a mortgage loan problem assume you are making constant payments on an amount PV which you have borrowed and on which you are charged an interest rate i (applied to the previous unpaid balance) over each of n equal time periods. Then the following equation holds where PMT is the amount of each periodic payment.

$$PV = PMT\left[\frac{1-(1+i)^{-n}}{i}\right]$$

There are 4 variables in this equation. If you borrow $5000 at 1% per month for 36 months you can use the above equation to solve for the amount of each payment. PMT = $166.07 This is a very straightforward problem to solve. A much more difficult problem is to try and solve the above equation for the interest rate i. If you borrow $8000 for 48 months and your monthly payments are $222.65, what monthly interest rate are you being charged?

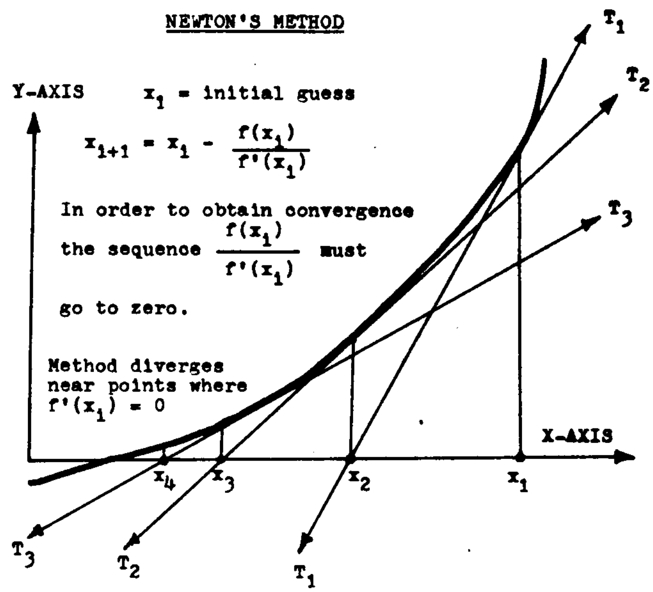
The above equation could be solved on paper for the variable PMT by anyone with a high school algebra background. But the second problem is impossible to solve for i using pencil & paper. About the best anyone could do would be to write the second problem in the form:

$$(222.65)\left[\frac{1-(1+i)^{-48}}{i}\right] - 8000 = 0$$

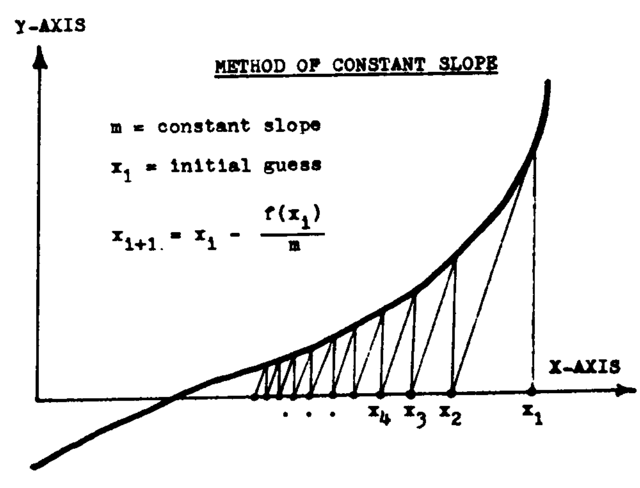
If we consider the left side of this equation as a function in the variable i, then this equation is of the form $f(i)=0$.

Most readers will probably be familiar with the Newton-Raphson Method which can be very effective in solving equations of the form $f(x)=0$ where algebraic methods fail. This article will give a simple but effective alternative to Newton's Method for solving these kinds of problems. Two other methods will also be mentioned but not discussed in detail.

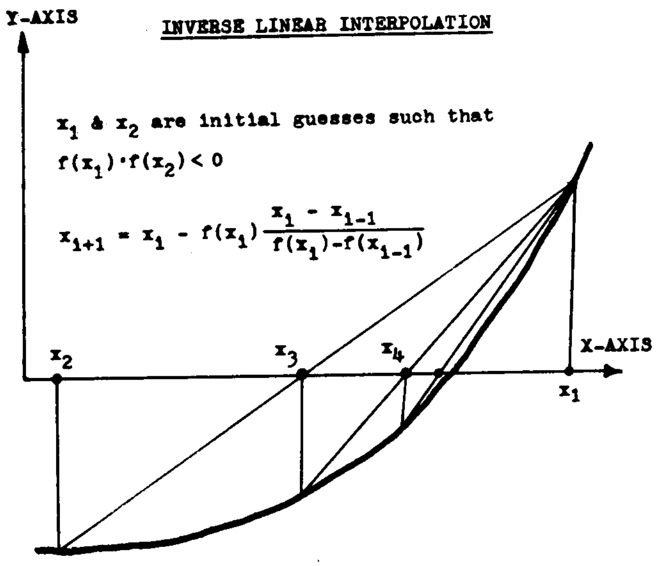
NEWTON'S METHOD works by estimating where the graph of the curve $y=f(x)$ crosses the x-axis. The tangent lines to the curve at successive points should then cross the x-axis at points closer and closer to the true root of the equation $f(x)=0$. The first derivative of the function $f(x)$ is used to obtain the slopes of the tangent lines.



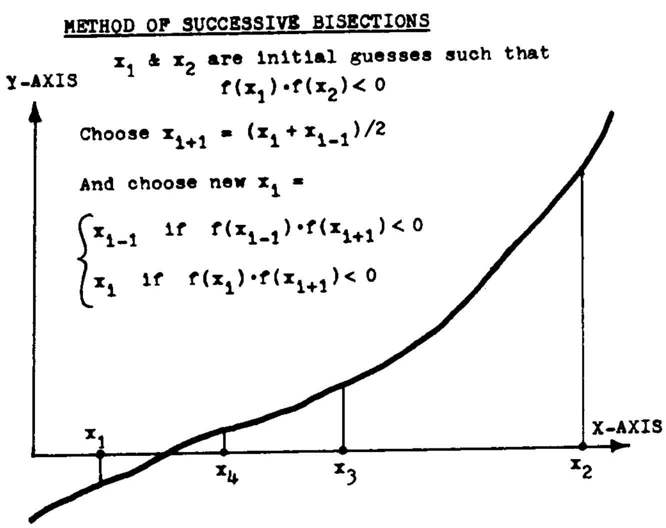
The METHOD OF CONSTANT SLOPE is very similar to Newton's Method but avoids the problem of computing the first derivative of the function $f(x)$. Instead, a constant slope m is used in the hope that secant lines to the curve at successive points will cross the x-axis at points closer and closer to the desired solution.



The METHOD OF INVERSE LINEAR INTERPOLATION (False Position) requires two starting points on the graph of $y=f(x)$, one above the x-axis and one below the x-axis. The straight line which connects these two points will then cross the x-axis in another point. The next point chosen on the graph corresponds to this intersection point on the x-axis. At each stage two points on the graph on opposite sides of the x-axis are maintained.



The METHOD OF SUCCESSIVE BISECTIONS also requires two starting points on the graph on opposite sides of the x-axis. The x-coordinates of these two points are then averaged to obtain the midpoint x-coordinate. This also determines a new point on the graph. The original point on the curve which is on the same side of the x-axis as this new point is then discarded. Thus the two points on the graph which are on opposite sides of the x-axis are kept.



All four methods given involve solving the equation $f(x)=0$ by successive approximations. Some methods will be better suited for some applications than others but I think the method of successive bisections is as close to a "universal method" as any. It is one of the simplest to apply and will always result in at least one solution.

While the fast convergence of Newton's Method makes it attractive, there are some difficulties with this method. Before briefly discussing these drawbacks I should mention that Newton's Method is an excellent choice whenever it applies. The fact is however that it doesn't always apply. It is less universal than successive bisections.

The first difficulty encountered with Newton's Method is computing the derivative. Historically this computation was carried out algebraically by hand on paper. While this can still be done and then programmed, in most computer applications the derivative is approximated numerically. $f'(x) \cong [f(x+\Delta x) - f(x)]/\Delta x$. Usually the numerical approximation doesn't present any difficulty but variations of the above formula have been developed to handle cases where such approximations lead to errors.

It is tempting to use the power of a computer to help you calculate the derivative. But Newton didn't have a computer to use. I maintain that because such calculations were done by hand it was mandatory that any method used had to result in fast convergence. But now that we have a tool with extremely fast computational speed, fast convergence due to the method used is no longer a requirement.

Newton's Method and successive bisections present a real contrast. Successive bisections would take an extremely long time if done by hand. But the method takes perfect advantage of computer power since a computer is used most efficiently when it performs one simple task over and over again. In this sense, successive bisections is better suited for computer solution than is Newton's Method.

Another problem is that Newton's Method technically applies only to differentiable functions. More serious is the problem that the method diverges near points where $f'(x)=0$ and near points of inflection where the graph of the function changes from concave up to concave down. Your initial guess in applying the method has to be good enough to avoid these problems.

The method of successive bisections is founded on a basic property of continuous functions. Whenever a continuous function takes on a positive and a negative value then it must also take on the intermediate value zero at some point in between the positive and negative points. For this reason, the positive and negative points bracket the zero solution.

Sometimes this method is called the method of interval halving. We compute the midpoint of the interval and test for a zero value there. If the function value at the midpoint is zero then we are done and otherwise the value there will be positive or negative. The important point is that at each stage we have trapped the true zero solution between a known positive and a known negative value. When we repeatedly halve successive intervals, the endpoints move closer together and after not too many steps the endpoints become numerically indistinguishable. If we don't have the true zero solution at that time then we will have found the one solution which comes closest to being zero.

The only restriction here is that we begin with a continuous function over some closed interval. Before we can apply successive bisections we must start with a positive and a negative point on the graph of the function. Finding these points is no more difficult than making a lucky first guess in Newton's Method. Once these conditions are met we do not have to worry about convergence. Even though it may take a minute or two longer to converge, the method of successive bisections always yields a useable solution.

The successive bisections program can be used to solve an equation of the form $f(x)=0$. The program requires two starting functional values of opposite sign. A small positive value ϵ is also required to determine when the interval endpoints are sufficiently close together to end the program.

INSTRUCTIONS:

1) GTO LBL E, switch to W/PRGM mode and key in the steps necessary to evaluate $f(x)$. Registers RA-RE are available to store constants associated with $f(x)$. End the routine with h RTN and switch back to RUN mode.
2) To find the two starting points key in a first value for x, say x_1, and press A. Then key in x_2 and press B. $f(x_1)$ & $f(x_2)$ must differ in sign. If $f(x_2)$ has the same sign as $f(x_1)$ then the display will flash all 0's to indicate that another value for x_2 must be tried. Key in a new x_2 and press B.
3) The value ϵ determines when the program ends if an exact zero solution is not found. The program will then end when $|x_i - x_{i-1}| \leq \epsilon$. Key in a value for ϵ (EEX CHS 8 will usually do) and press C.
4) Press D. The program will stop and display zero if $f(x)=0$. Otherwise the program will display

the x value with blinking decimal point and then stop and display f(x) which will be near zero. In either case x remains stored in R5 and f(x) is stored in R6.

To evaluate f(x) at any x, key in x and press E.

EXAMPLE: Let's solve the previous loan problem.

$$f(i) = (222.65)\left[\frac{1-(1+i)^{-48}}{i}\right] - 8000 = 0$$

GTO E and key in the function as: STO D, 1, +, RCL A, h y^x, 1, -, CHS, RCL D, ÷, RCL B, X, RCL C, -, h RTN. Then store -48 in RA, store 222.65 in RB and store 8000 in RC.
Key .005 for x_1 and press A. Key .5 for x_2 and press B.
Key EEX CHS 8 and press C. Press D to solve. The program should display i = 0.012500826 with blinking decimal point and then stop and display f(i) = -0.000206000. The monthly interest rate i ≅ 1.25%.

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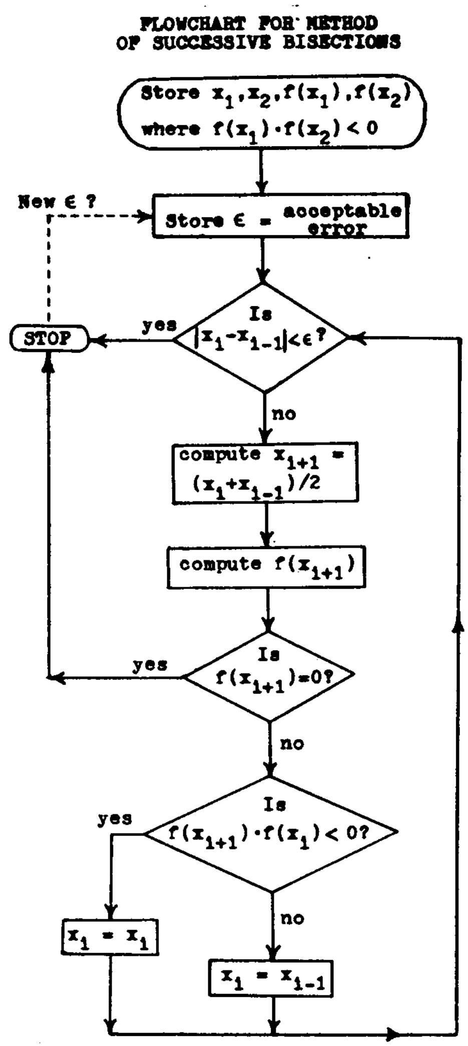
FLOWCHART FOR METHOD OF SUCCESSIVE BISECTIONS



R/S

SUCCESSIVE BISECTIONS — John Kennedy (918)

1	1st Point	2nd Point	ε	Solve f(x)=0	Evaluate f(x)	2

HP-67/97 PROGRAM SUCCESSIVE BISECTIONS By JOHN KENNEDY

001	*LBLA	21 11	022	RTN	24	043	*LBL1	21 01
002	STO1	35 01	023	*LBLD	21 14	044	RCL5	36 05
003	GSBE	23 15	024	RCL1	36 01	045	STO3	35 03
004	STO2	35 02	025	RCL3	36 03	046	RCL6	36 06
005	RTN	24	026	+	-55	047	STO4	35 04
006	*LBLB	21 12	027	2	02	048	*LBL2	21 02
007	STO3	35 03	028	÷	-24	049	RCL0	36 00
008	GSBE	23 15	029	STO5	35 05	050	RCL3	36 03
009	STO4	35 04	030	GSBE	23 15	051	RCL1	36 01
010	RCL2	36 02	031	STO6	35 06	052	-	-45
011	x	-35	032	X=0?	16-43	053	ABS	16 31
012	X>0?	16-44	033	RTN	24	054	X>Y?	16-34
013	GTO0	22 00	034	RCL2	36 02	055	GTOD	22 14
014	RCL4	36 04	035	x	-35	056	RCL5	36 05
015	RTN	24	036	X<0?	16-45	057	PRTX	-14
016	*LBL0	21 00	037	GTO1	22 01	058	RCL6	36 06
017	0	00	038	RCL5	36 05	059	RTN	24
018	PSE	16 51	039	STO1	35 01	060	*LBLE	21 15
019	GTO0	22 00	040	RCL6	36 06	061	RTN	24
020	*LBLC	21 13	041	STO2	35 02	062	R/S	51
021	STO0	35 00	042	GTO2	22 02			

HEX TO/FROM DECIMAL - 29C

HEX TO DECIMAL - 29C

01	15 13 03	g LBL3	13	12 04	GOSUB4
02	24 02	RCL2	14	24 01	RCL1
03	12 04	GOSUB4	15	15 12	g RTN
04	24 03	RCL3	16	15 13 04	g LBL4
05	12 04	GOSUB4	17	24 00	RCL0
06	24 04	RCL4	18	61	X
07	12 04	GOSUB4	19	23 51 01	STO + 1
08	24 05	RCL5	20	01	1
09	12 04	GOSUB4	21	06	6
10	24 06	RCL6	22	23 61 00	STO X 0
11	12 04	GOSUB4	23	15 12	g RTN
12	24 07	RCL7			

1. CLR REG
2. Store each digit beginning with unit digit in R1, tens digit in R2, hundreds digit in R3, etc. Maximum 7 entries. If "digit" is alpha, store proper number equivalent from Table. See example.
3. STO 16 R0
4. GSB3
5. Display will show decimal number.

Example: Hex BC614E STORE as follows:

STO	14	(E)	Unit	R1	
STO	4		Tens	R2	After Steps 1-2-3
STO	1		Etc	R3	Dispaly will show
STO	6			R4	12345678
STO	12	(C)		R5	
STO	11	(B)		R6	

Example: Hex 8000 STORE as follows:

STO	0	Unit	R1	After Steps 1-2-3
STO	0	Tens	R2	Display will show
STO	0	ETC	R3	32768
STO	8		R4	

DECIMAL TO HEX - 29C

01	15 13 01	g LBL1	14	23 22	STOi
02	23 .3	STO.3	15	01	1
03	24 .1	RCL.1	16	23 51 .5	STO +.5
04	71	÷	17	24 .4	RCL.4
05	14 62	f INT	18	15 24	ISZ
06	23 .4	STO.4	19	13 01	GOTO1
07	24 .1	RCL.1	20	15 13 02	g LBL2
08	61	X	21	24 .3	RCL.3
09	24 .3	RCL.3	22	23 22	STOi
10	21	x ≷ y	23	01	1
11	15 71	g X=0	24	23 51 .5	STO +.5
12	13 02	GOTO2	25	24 .5	RCL.5
13	41	-	26	15 12	g RTN

1. CLR REG
2. STO 16 R.1 STO 1 R0
3. Enter decimal number
4. GSB1
5. Hex digits are in Registers 1 thru 9. List R1 as unit digit, R2 as tens digit, R3 as hundreds digit, etc. For numbers greater than 9 convert to alpha from Table. Discontinue Register recall after Register number corresponding to number in Display.

Example: Decimal 12345678 to Hex

RCL	R1	14	(E)	Unit	E	
RCL	R2	4		Tens	4	
RCL	R3	1		Etc	1	
RCL	R4	6			6	Hex: BC614E
RCL	R5	12	(C)		C	
RCL	R6	11	(B)		B	

No further recall as "6" in display indicates R6 to be final Register for recall.

Example: Decimal 32768 to Hex